Appendix S1

We derive here some asymptotic properties of model (5) but we will use Verhulst's r-α formulation of the logistic model because it makes mathematical derivation lighter. The usual r-K parameterization is obtained with $K = r/\alpha$.

Firstly, we prove that under perfect mixing ($\beta \to \infty$) we have the following equality:

$$\frac{N_1}{\gamma_1} = \frac{N_2}{\gamma_2}.$$

For that, we use Eq. (5) to consider the following difference:

$$\frac{d}{dt}\left(\frac{N_1}{\gamma_1} - \frac{N_2}{\gamma_2}\right) = \frac{1}{\gamma_1}\frac{dN_1}{dt} - \frac{1}{\gamma_2}\frac{dN_2}{dt} = \frac{r_1}{\gamma_1}N_1 - \frac{\alpha_1}{\gamma_1}N_1^2 - \frac{r_2}{\gamma_2}N_2 + \frac{\alpha_2}{\gamma_2}N_2^2 - \beta\left(\frac{1}{\gamma_1} + \frac{1}{\gamma_2}\right)\left(\frac{N_1}{\gamma_1} - \frac{N_2}{\gamma_2}\right).$$

In the limit $\beta \to \infty$, the last term in the RHS becomes predominant and the equation tends to:

$$\frac{d}{dt}\left(\frac{N_1}{\gamma_1} - \frac{N_2}{\gamma_2}\right) = -\beta\left(\frac{1}{\gamma_1} + \frac{1}{\gamma_2}\right)\left(\frac{N_1}{\gamma_1} - \frac{N_2}{\gamma_2}\right).$$

Consequently $\frac{N_1}{\gamma_1} - \frac{N_2}{\gamma_2}$ tends to zero, that is $\frac{N_1}{\gamma_1} = \frac{N_2}{\gamma_2}$. From this equality, we can also deduce that

$$\frac{N_T}{\gamma_1 + \gamma_2} = \frac{N_1 + N_2}{\gamma_1 + \gamma_2} = \frac{N_1\frac{\gamma_1}{\gamma_1} + N_2\frac{\gamma_2}{\gamma_2}}{\gamma_1 + \gamma_2} = \frac{N_1}{\gamma_1} = \frac{N_2}{\gamma_2}. \tag{A1}$$

Secondly, we calculate the dynamics of total abundance. Equations (5) modified in the r-α formulation give:

$$\frac{dN_T}{dt} = \frac{dN_1}{dt} + \frac{dN_2}{dt} = r_1N_1 - \alpha_1N_1^2 + r_2N_2 - \alpha_2N_2^2$$

and using (A1), we have:

$$\frac{dN_T}{dt} = r_1 N_1 \frac{\gamma_1}{\gamma_1} - \alpha_1 N_1^2 \left(\frac{\gamma_1}{\gamma_1}\right)^2 + r_2 N_2 \frac{\gamma_2}{\gamma_2} - \alpha_2 N_2^2 \left(\frac{\gamma_2}{\gamma_2}\right)^2$$
$$= \left(r_1\gamma_1 + r_2\gamma_2\right)\frac{N_1}{\gamma_1} - \left(\alpha_1\gamma_1^2 + \alpha_2\gamma_2^2\right)\left(\frac{N_1}{\gamma_1}\right)^2$$
$$= \underbrace{\frac{r_1\gamma_1 + r_2\gamma_2}{\gamma_1+\gamma_2}}_{\equiv \bar{r}} N_T - \underbrace{\frac{\alpha_1\gamma_1^2 + \alpha_2\gamma_2^2}{\left(\gamma_1+\gamma_2\right)^2}}_{\equiv \alpha_T} N_T^2$$

We can now return to the r-K parameterization and calculate:

$$K_T = \frac{\bar{r}}{\alpha_T} = \frac{\left(r_1\gamma_1 + r_2\gamma_2\right)\left(\gamma_1+\gamma_2\right)}{\alpha_1\gamma_1^2 + \alpha_2\gamma_2^2}, \tag{A2}$$

which can be rewritten as in (6) in order to isolate $K_1 + K_2$.

Additivity of carrying capacities ($K_T = K_1 + K_2$) occurs when the numerator of the fraction in (6) is zero. This leads to a second-degree equation in the unknown $x = \gamma_1 / \gamma_2$, which has the following two solutions:

$$\frac{\gamma_1}{\gamma_2} = \frac{K_1}{K_2}, \tag{A3}$$

$$\frac{\gamma_1}{\gamma_2} = \frac{r_2/K_2}{r_1/K_1} = \frac{\alpha_2}{\alpha_1}. \tag{A4}$$

Recalling the definitions $\beta_i = \beta / \gamma_i$, the two occurences of additivity are therefore:

$$\beta_1 = \frac{\beta}{K_1} \text{ and } \beta_2 = \frac{\beta}{K_2}, \tag{A5}$$

$$\beta_1 = \beta\alpha_1 \text{ and } \beta_2 = \beta\alpha_2. \tag{A6}$$