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Gross versus net equalization scheme in a federation with decentralized leadership

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ABSTRACT

This paper compares the impact of two equalization transfer schemes on regional budgetary choices: a gross equalization scheme, where *ex-post* transfers to regions are financed from federal tax revenues, and a net equalization scheme, where region-to-region ex-post transfers allocated by the federal government are self-financed. The net equalization scheme reduces ex-post federal intervention and should therefore be favored over the gross equalization scheme in order to reduce opportunistic regional behaviour. In addition, the two equalization schemes differently affect the composition of public expenditures. The gross equalization scheme generates an upward distortion of the amount of capital public expenditures whereas the net equalization scheme generates a downward distortion of the amount of current public expenditures compared to the optimum. A welfare analysis reveals that incentives to use a net equalization scheme are strengthened for a sufficiently high number of regions.

1. Introduction

Fiscal equalization is a necessary complement to decentralization to reduce fiscal disparities among subnational jurisdictions. Although we observe a great variety in equalization schemes across OECD countries (Blöchliger et al., 2007), two broad forms of equalization can be identified according to the terminology of Boadway (2004, 2007).[1] Under a gross equalization scheme, the federal government raises tax revenues to finance ex-post transfers to regions. Under a net equalization scheme, ex-post transfers are self-financed by regions, which implies that positive ex-post transfers to some regions are financed by negative ex-post transfers to the other regions. As pointed out by Dafflon (2007), the gross equalization scheme has been widely implemented whereas only few countries (e.g. Australia and Denmark) have adopted a net equalization scheme to reduce disparities in expenditures. However, to our knowledge, theoretical justifications for giving one equalization scheme an advantage over the other are lacking, although different structures of equalization are likely to have different influences over regional budgetary policies.

The equalization policy can result in perverse incentives for regional governments if they rationally anticipate how the equalization transfers will respond to their policy changes (Weingast, 2009). In particular, constitutional reasons, e.g. equalization of living conditions wherever citizens are located like in Germany, or fixed rules of the equalization schemes allow the regions to anticipate the determinants of the allocation of federal transfers. These expectations may induce regional governments to reduce their tax efforts or to increase their expenditures, with the aim of extracting more transfers, as pointed out by Wurzel (2003) and the OECD (2006). Regional decentralized leadership is particularly appropriate for characterizing intergovernmental relationships when regional governments are first-movers.[2] Although a net equalization

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[1] For a comprehensive analysis on design and worldwide practices in intergovernmental fiscal transfers, see Boadway (2007).

[2] The decentralized leadership concept is closely related to the soft budget constraint concept. Besides, the soft budget constraint issue is usually also formulated in the context of a sequential game where the first move is made by regional governments; the federal government has the second move and at that point the costs to the federal government of not providing additional funds may exceed these of bailing out (Wildasin, 1997; Garcia-Mila et al., 2002; Goodspeed, 2002; Kornai et al., 2003; Büttner, 2006; Breuillé et al., 2006; Akai and Silva, 2009; Breuillé and Vigneault, 2010). Note that a hard budget constraint is not always optimal because it can discourage investment that is socially efficient, as shown by Besfamille and Lockwood (2008). Empirical evidence suggests that the soft budget constraint phenomenon induces selective (and usually rare) bailouts from the federal government (see Rodden et al. (2003) and Vigneault (2007) for a survey) which contrasts with generalized (and systematic) ex-post equalizing transfers with decentralized leadership. In addition, contrary to soft budget constraint models, we rule out time inconsistency in this paper.

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scheme seems to be less prone to opportunistic regional behaviour than a gross equalization scheme, the respective impact of both equalization schemes on welfare is uncertain at first sight.

Our paper compares these two equalization schemes, i.e. the gross equalization scheme versus the net equalization scheme, under decentralized leadership, with the aim of determining which equalization scheme maximizes welfare countrywide. Regions finance both current public expenditures, e.g. recreational facilities or social services, which enter the citizens' utility and capital public expenditures, e.g. transportation expenditures, which benefit firms.[3] These public expenditures are financed by both a regional tax on mobile capital as well as transfers. While Keen and Marchand (1997) show how competition for mobile capital leads to systematic distortions of the pattern of public expenditures in the absence of federal intervention,[4] we go further by investigating how these distortions of the pattern of public expenditures are affected by equalization schemes implemented by the federal government. In a simultaneous-move game, regions compete through both their tax rate and capital expenditures to attract mobile capital, as in Keen and Marchand (1997), and they also compete to attract ex-post federal transfers. However, taxes on elastically supplied labour and pure profits are absent in our model.

The basic set-up is a one-period model with decentralized leadership, similar to Köthenbürger (2004) and Akai and Sato (2008), in which the federal government moves after observing choices of regional governments. Regional governments act as Stackelberg leaders, i.e. have a strategic advantage, vis-à-vis the federal government, in the transfer game. In Köthenbürger (2004), transfers to regions are granted by the federal government according to a net equalization scheme and only current public expenditures are financed. Although the net equalization scheme insulates regions from harmful tax competition, Köthenbürger (2004) shows that the federal intervention may reduce welfare compared to the absence of federal intervention, due to the perverse incentive generated by the common pool effect. We clearly depart from his analytical framework, firstly by comparing the net equalization scheme with the gross equalization scheme, and secondly by considering that both current and capital public expenditures are financed by regions.[5] Our model differs from Breuillé and Vigneault (2010) for the same reasons because their model of regional decentralized leadership is similar to Köthenbürger (2004), with the important difference that they consider a third tier. This leads to an overlapping soft budget constraint, which worsens the soft budget constraint issue: the softer the regional budget constraint, the softer the local budget constraint. They also demonstrate that horizontal tax competition among the rescuers does not harden the budget constraint, contrary to the result of Qian and Roland (1998). We also depart noticeably from Akai and Sato (2008), whose contribution is to determine how different scenarios in regional decision-making – namely whether public expenditure or tax collection is decided ex-ante – affect the equilibrium outcome. Their scenario B is closer to our model, as current public expenditures are provided as residual to fulfill the regional budget constraint. Like Köthenbürger (2004), they assume that transfers are allocated according to a net equalization scheme and only current public expenditures are financed. Their model mainly differs from Köthenbürger (2004) and from our work by the absence of tax competition among regions, by the asymmetry among regions and by the existence of spillover effects. Admittedly, symmetry is a stark assumption, but it allows us to focus exclusively on the incentive effects of ex-post transfers on regional budgetary policy, without any redistributive effects.

In all these papers that assume decentralized leadership, including ours, the rationale for equalization – like in most countries (Blöchliger et al., 2007) – is equity in the sense of having equal access to public services across regions (Oakland, 1994; Boadway, 2001). The horizontal equity objective that our utilitarian federal government faces requires the equalization of marginal utilities from current public expenditures across regions, which implies an equal amount of current public expenditures due to our setting. As a consequence, equals are treated equally. To reach this aim of horizontal equity, we alternately use a gross and a net equalization schemes. Given our formalization, these two schemes differentiate not only by the origin of ex-post transfers, i.e. financed by the federal government or self-financed by regions themselves, but also by the distortive nature of tax revenues that are raised. Under the gross equalization scheme, ex-post transfers are entirely financed by the federal government through a non-distortive lump-sum tax, whereas under the net equalization scheme, ex-post transfers are self-financed by regions through a distortive tax on mobile capital.

As first movers, regions anticipate that ex-post transfers are allocated so as to provide an equal access to public services. Ex-post transfers thus depend on ex-ante decisions, i.e. on tax rates and on capital public expenditures, which make transfers conditional upon these two budgetary choices. This game with sequential moves played by the benevolent center and selfish regions can be related to the rotten kid literature. Essentially, the federal government is a "benevolent parent" who offers gifts of money to its "rotten regions". The rotten kid theorem, first stated by Becker (1974, 1981), applies when selfish kids internalize externalities within the family and implement the parent's most preferred income, in the presence of parental transfers, even if the parent cannot precommit to an incentive scheme. Caplan et al. (2000) and Akai and Silva (2009) both show that the rotten kid theorem also applies in the fiscal federalism literature.[6] However, their results are attributable to specific assumptions[7] and cannot be generalized to all papers that deal with an interregional redistribution issue in a federation characterized by regional decentralized leadership. In our paper, "rotten regions" do not behave well because there is no conditional transferable utility (Bergstrom, 1989), which is due to the absence of linearity of the regional utility with respect to the transfer granted by the federal government. Indeed, transfers are devoted to the financing of current public expenditures, which are valued through a concave function. Our subgame perfect equilibrium under the decentralized leadership thus clearly differs from the Pareto optimum. More generally, for the rotten kid theorem to hold, the prices of the selfish agents' payoffs to the benevolent contributor must be beyond manipulation, as proved by Dijkstra (2007), which is not the case in our paper.

Four main results emerge from our paper. First, we show that the net equalization scheme is less prone to opportunistic regional behaviour than a gross equalization scheme, as the amount of

[3] See Zodrow and Mieszkowski (1986), Matsumoto (1998), Bayindir-Upmann (1999) for papers on public input provision.

[4] Matsumoto (2008) extends Keen and Marchand (1997) by showing that the expenditure mix is inefficient – i.e. too much is spent on development policies and too little on transfer policies – in a different setting with both regional disparities and a different production technology.

[5] There are also a number of important distinctions with Breuillé et al. (2006) who consider, in a two-period model with regional debt, that federal transfers are financed by a distortive tax on capital and help regions to increase their current public expenditures. Here, we rule out regional debt and exclude vertical tax competition as ex-post transfers are either self-financed or financed via a federal lump-sum tax.

[6] Efficiency can also be achieved in the absence of a central/federal government in models with citizens mobility (Myers, 1990; Mansoorian and Myers, 1993).

[7] In Caplan et al. (2000) – following Cornes and Silva (1999) – the rotten kid theorem applies because of the perfect substitutability of contributions to the single pure public good. As for Akai and Silva (2009), the rotten kid theorem applies in their model with asymmetric information because of: (i) the existence of ex-ante transfers and (ii) the linearity of the regional utility with respect to the transfer granted by the central government. Consequently, the regional government's objective coincides perfectly with the federal government's objective.

ex-post transfers is reduced. Self-financing region-to-region transfers make each region more accountable because its representative citizen bears a higher cost.

Secondly, we derive original results from the analysis of public expenditures. The pattern of public expenditures crucially depends on the equalization transfer scheme at work: the decentralized leadership equilibrium with a gross equalization scheme is characterized by an optimal amount of current public expenditures and an upward distortion of the amount of capital public expenditures, whereas the decentralized leadership equilibrium with a net equalization scheme is characterized by a downward distortion of the amount of current public expenditures and an optimal amount of capital public expenditures. In a nutshell, ex-post gross equalization entirely compensates fiscal externalities arising with capital mobility without distorting the marginal cost for public funds, whereas ex-post net equalization, due to revenue sharing, distorts the marginal cost for public funds.

Thirdly, we extend the result derived in a one-tier setting by Keen and Marchand (1997) to a two-tier setting with decentralized leadership. At the non-cooperative equilibrium, we demonstrate that welfare would be improved ceteris paribus by a coordinated reduction in capital public expenditures and a corresponding increase in current public expenditures, a fortiori under a net equalization scheme.

Finally, welfare analysis reveals that incentives to use a net equalization scheme are strengthened for a sufficiently high number of regions.

The paper is organized as follows: Section 2 presents the two-tier framework of the model; Section 3 derives the outcome of the first-best solution and the solution without decentralized leadership, which will both serve as benchmarks for comparison purposes; the effects on federal and regional decisions of the gross versus net equalization scheme with decentralization leadership are presented and then compared in Section 4; and Section 5 concludes.

2. The framework

The federation is composed of n identical regions. Each region i (for $i = 1,\ldots,n$) finances both current public expenditures (e.g. public goods) in quantity G_i and capital public expenditures (e.g. public inputs) in quantity I_i.

2.1. The representative household

Let $U(c_i, G_i)$ be the utility the representative household of the region i derives from the provision of public goods, G_i, and from the consumption of a private good, denoted by c_i:

$$U(c_i, G_i) = c_i + v(G_i), \tag{1}$$

where the utility function $v(\cdot)$ is increasing in its argument, twice differentiable and strictly concave. This representative household is initially endowed with $\widetilde{K}$ units of capital, i.e. $\frac{1}{n}$ of the capital available in the federation, that are remunerated at the before-tax interest rate r_i on which a regional tax τ_i is levied according to the source principle.[8] A unique firm is located in each region i and owned by the representative household, who receives the profit, Π_i. Private consumption thus amounts to the profit of the firm, plus the remuneration of the initial capital endowment at the net return $\rho = r_i - \tau_i$, minus a lump-sum tax Γ levied by the federal government.[9] The representative household's budget constraint is given by:

$$c_i = \Pi_i + \rho\widetilde{K} - \Gamma. \tag{2}$$

2.2. The capital market

The firm located in region i produces an output $F(K_i, I_i)$ from the capital, K_i, borrowed on the market at the interest rate r_i and from the capital public expenditures,[10] I_i. The production function is monotonously increasing in both production factors $\left(\frac{\partial F}{\partial K_i} \equiv F_{K_i} > 0 \text{ and } \frac{\partial F}{\partial I_i} \equiv F_{I_i} > 0\right)$ and has decreasing marginal products. Based on a review of empirical evidences, we assume that capital public expenditures increase the marginal productivity of capital $\left(\frac{\partial^2 F}{\partial K_i \partial I_i} \equiv F_{K_i I_i} > 0\right)$, due to a complementary between the two production factors.[11] The profit of the firm located in region i, which totally accrues to the representative household is given by:

$$\Pi_i = F(K_i, I_i) - r_i K_i.$$

Firms' profit maximizing behavior implies the familiar condition of remuneration at the marginal productivity of capital, $\frac{\partial F}{\partial K_i} = r_i \ \forall\ i$. The resulting demand for capital $K_i(r_i, I_i)$ and profit $\Pi_i(r_i, I_i)$ are decreasing functions of the interest rate r_i, i.e. $\frac{\partial K_i(\cdot)}{\partial r_i} \equiv K_{r_i} = \frac{1}{F_{K_i K_i}} < 0$ and $\frac{\partial \Pi_i(\cdot)}{\partial r_i} \equiv \Pi_{r_i} = -K_i < 0$, and increasing functions of the capital public expenditures, i.e. $\frac{\partial K_i(\cdot)}{\partial I_i} \equiv K_{I_i} = -\frac{F_{K_i I_i}}{F_{K_i K_i}} > 0$ and $\frac{\partial \Pi_i(\cdot)}{\partial I_i} \equiv \Pi_{I_i} = F_{I_i} > 0$. The capital market clearing condition in the federation,

$$\sum_{i=1}^{n} K_i(r_i, I_i) = n\widetilde{K},$$

characterizes the capital market equilibrium, i.e. it defines in a symmetric setting the net return $\rho(\tau_1,\ldots,\tau_n, I_1,\ldots,I_n)$ as a decreasing function of the regional tax rate:

$$\frac{\partial \rho}{\partial \tau_i} = -\frac{\frac{\partial K_i}{\partial r_i}}{\sum_{i=1}^{n} \frac{\partial K_i}{\partial r_i}} = -\frac{1}{n} \qquad \in\ ]-1, 0[,$$

and as an increasing function of the capital public expenditures:

$$\frac{\partial \rho}{\partial I_i} = -\frac{\frac{\partial K_i}{\partial I_i}}{\sum_{i=1}^{n} \frac{\partial K_i}{\partial r_i}} = -\frac{\frac{\partial K_i}{\partial I_i}}{n\frac{\partial K_i}{\partial r_i}} > 0,$$

which implies that the interest rate moves as follows:

$$\frac{\partial r_i}{\partial \tau_i} = 1 + \frac{\partial \rho}{\partial \tau_i} = \frac{n-1}{n} \quad \in\ ]0, 1[\qquad \text{and} \qquad \frac{\partial r_i}{\partial \tau_{-i}} = \frac{\partial \rho}{\partial \tau_{-i}} = -\frac{1}{n} \quad \in\ ]-1, 0[.$$

A rise in the amount of capital public expenditures thus produces two opposite effects on both the demand for capital and the profit: (i) a direct positive effect ($K_{I_i} > 0$ and $\Pi_{I_i} > 0$) and (ii) an indirect negative effect through the net return on capital ($K_{r_i}\frac{\partial \rho}{\partial I_i} < 0$ and $\Pi_{r_i}\frac{\partial \rho}{\partial I_i} < 0$). Note that $\tau_i K_{r_i}\frac{\partial \rho}{\partial I_i}$ can be seen as a "crowding-out effect" of public investment on private capital: a rise in capital public expenditures increases the net return of capital (and thus the interest rate) which in turn dissuades private investment.

Finally, in line with empirical findings,[12] we postulate that the elasticity of the regional tax base with respect to the regional tax

[8] There are two polar principles of taxation: the residence (of the taxpayer) principle and the source (of income) principle. The source principle implies that all incomes originating in a region are taxed in that region regardless of the residence of the taxpayers.

[9] Capital relocates until it earns the same net return of capital in each region, i.e. $\rho_i = \rho_j = \rho\ \forall\ i,j$.

[10] In reference to Matsumoto (1998), I_i is a "factor-augmenting public input". The production function exhibits CRS in private inputs.

[11] Empirical evidence on aggregate production relationships suggests that public infrastructure has almost always been found to be complementary with private capital (Sturm et al., 1996).

[12] See Chirinko et al. (1999) for instance.

rate, denoted by $\varepsilon_i \equiv \frac{\partial K_i}{\partial \tau_i}\frac{\tau_i}{K_i}\ \forall\ i$ belongs to the interval $]-1, 0[$ which implies $\frac{\partial \tau_i K_i}{\partial \tau_i} = (1+\varepsilon_i)K_i > 0$.

2.3. Federal and regional governments

Both federal and regional governments are benevolent. Each regional government acts so as to maximize the utility of the representative household located in its region. The budget constraint of the region i is expressed by:

$$I_i + G_i = T_i + \tau_i K_i(r_i, I_i), \tag{3}$$

where T_i is a federal transfer. These transfers are allocated by the federal government to regions so as to maximize the aggregated utility $\sum_{i=1}^{n} U(c_i, G_i)$. All regions count equally in the welfare function. An horizontal equity requirement results from the problem the utilitarian federal government faces when it makes interregional equalization. The federal government is averse to inequality in terms of regional public good provision because of the concavity of $v(\cdot)$.

In what follows, we alternately consider two broad forms of redistribution among regions, according to the distinction proposed by Boadway (2004, 2007). Firstly, a "gross equalization scheme" where all transfers to regions are financed from federal tax revenues: a lump-sum tax Γ levied on each household is endogenously chosen by the federal government. Therefore, every additional transfer to one region, hereafter called an ex-post transfer, is financed by additional federal tax revenues, that is $\sum_{i=1}^{n} dT_i = n\,d\Gamma$. The federal budget constraint under a gross equalization scheme writes:

$$\sum_{i=1}^{n} T_i = n\Gamma. \tag{4}$$

Secondly, a "net equalization scheme", where the federal government has no ability to modify the size of the equalization pool: $n\overline{\Gamma}$ is exogenous and the parameter $\overline{\Gamma}$ belongs to the interval $[0, \gamma]$ with $\gamma > 0$. This implies that region-to-region ex-post transfers allocated by the federal government are self-financed. Therefore, every additional transfer to one region must be compensated by contributions made by the other regions, that is $\sum_{i=1}^{n} dT_i = 0$. The federal budget constraint under a net equalization scheme writes:

$$\sum_{i=1}^{n} T_i = n\overline{\Gamma}. \tag{5}$$

In a way, a gross equalization scheme means that the size and the cutting of the cake "transfers to regions" are both determined by the federal government, whereas a net equalization scheme means that only the rules for dividing the cake – whose size is fixed – are determined by the federal government.

3. Benchmark cases

Before we proceed to the analysis of the game with decentralized leadership, we first present the outcome of the first-best solution and the outcome of the Nash game, which will both serve for comparison purposes.

3.1. The first-best solution

In this benchmark case, a benevolent social planner takes all the budgetary decisions so as to maximize the aggregated welfare $\sum_i U(c_i, G_i)$ subject to the budget constraints (2), (3) and (4) or (5). These choices perfectly internalize the budgetary externalities linked to fiscal competition among regions. At the first-best optimum, whichever the form of the equalization scheme, the transfers to regions $\{T_i\}_{i=1,\ldots,n}$ equalize the marginal utilities from the current public expenditures; i.e. $\frac{\partial v}{\partial G_i} = \frac{\partial v}{\partial G_k} = v'\ \forall\ i,k$. Like Zodrow and Mieszkowski (1986), both current and capital public expenditures are optimally provided; formally, $F_{I_i} = v'(G_i) = 1\ \forall\ i$.

3.2. The solution without regional decentralized leadership (Nash game)

In this benchmark case, the federal government and the regional governments simultaneously select their budgetary choices. Due to the absence of decentralized leadership, the regional governments are unable to manipulate their choices in order to attract ex-post federal transfers, that is $\frac{\partial T_i}{\partial \tau_i} = \frac{\partial T_i}{\partial I_i} = 0\ \forall\ i$. The transfers to regions $\{T_i\}_{i=1,\ldots,n}$, chosen by the federal government to maximize the aggregated welfare $\sum_i U(c_i, G_i)$, equalize the marginal utilities from the current public expenditures; i.e. $\frac{\partial v}{\partial G_i} = \frac{\partial v}{\partial G_k}\ \forall\ i,k$. Each region i simultaneously chooses τ_i and I_i in order to maximize the regional welfare $U(c_i, G_i)$; i.e. $v'(G_i) = \frac{1}{(1+\varepsilon_i)}$ and $F_{I_i} = v'[1 - \tau_i K_{I_i} - \tau_i K_{r_i}\frac{\partial \rho}{\partial I_i}] = v'\left[1 - \tau_i K_{I_i}\left(1 - \frac{1}{n}\right)\right]$. The amount of current public expenditures is determined as residual to fulfill the regional budget constraint. An interior solution requires that $1 - \tau_i K_{I_i} - \tau_i K_{r_i}\frac{\partial \rho}{\partial I_i} \geqslant 0$, which we assume subsequently.[13] This assumption means that a decrease in the capital public expenditures increases the current public expenditures ceteris paribus, i.e. for T_i given.

FOCs under a net equalization scheme without regional decentralized leadership are equivalent to the ones without federal intervention. As it is well-known in the literature, the capital tax competition results in an underprovision of the public good, i.e. too little current public expenditures. Similarly, the amount of capital public expenditures is sub-optimal due to fiscal externalities. However, we are unable to determine whether capital public expenditures are upwardly or downwardly distorted without specifying the production technology.[14]

By contrast, under a gross equalization scheme, the federal government sets its lump-sum tax so that $\frac{\partial v}{\partial G_i} = \frac{\partial v}{\partial G_k} = 1\ \forall\ i,k$, which restores the first-best solution. Note that it corresponds to the first-best case in Keen and Marchand (1997) in which regional governments are able to deploy a lump-sum tax.

4. Equalization with regional decentralized leadership

We now introduce decentralized leadership and analyze the impact of a gross versus net equalization scheme on regional budgetary incentives.

4.1. The timing of the game

Both federal and regional governments are assumed to be benevolent. Regional governments act as Stackelberg leaders vis-à-vis the federal government, which gives rise to a standard two-stage game (Köthenbürger, 2004; Akai and Sato, 2008) between the two levels of government:

> Stage 1 (ex-ante): Regional governments simultaneously choose their capital tax rates $\{\tau_i\}_{i=1,\ldots,n}$ and capital public expenditures $\{I_i\}_{i=1,\ldots,n}$, taking into account the reaction of the federal government. In doing so, they play as Nash competitors vis-à-vis each other, but as Stackelberg leaders vis-à-vis the federal government.

[13] We assume that $F(K_i, I_i)$ has the good properties to ensure that $1 - \tau_i K_{I_i} - \tau_i K_{r_i}\frac{\partial \rho}{\partial I_i} \geqslant 0$.

[14] If we assume that $1 - K_i F_{K_i I_i} > 0$, like Zodrow and Mieszkowski (1986) and Keen and Marchand (1997), capital public expenditures are downwardly distorted.

Stage 2 (ex-post): Given the regional budgetary decisions, the federal government allocates transfers $\{T_i\}_{i=1,\ldots,n}$ to regions. Furthermore, it also chooses the federal lump-sum tax Γ if a gross equalization scheme is implemented. Ex-post transfers thus depend on ex-ante decisions, i.e. on tax rates and on capital public expenditures, which makes transfers conditional to these two budgetary choices. Current public expenditures $\{G_i\}_{i=1,\ldots,n}$ are determined as residual after federal transfers are made.

Given these policy choices, firms determine capital demand, and then production takes place. Finally, each household receives profits from his/her firm's activity and enjoys the consumption of both private and public goods. These two last stages are implicitly introduced in our analysis. Federal and regional governments take into account the reaction of the capital demand when making their budgetary decisions and households' preferences guide the choices of the governments as they are both benevolent.

The sub-game-perfect equilibrium is determined by solving the governments' choice problems backwards, i.e. from the federal to the regional governments.

4.2. The federal government's problem

4.2.1. With a gross equalization scheme

The federal government chooses both the amount of aggregated transfers, or indifferently the value of the lump-sum tax Γ, and the division of transfers among regions, to maximize the aggregated utility of households in the federation. It solves:

$$\max_{\mathbf{T},\Gamma} \sum_{i=1}^{n}[c_i + v(G_i)], \tag{6a}$$

$$\text{s.t. } c_i = \Pi_i(r_i, I_i) + \rho\widetilde{K} - \Gamma, \tag{6b}$$

$$I_i + G_i = T_i + \tau_i K_i(r_i, I_i), \tag{6c}$$

$$\sum_{i=1}^{n} T_i = n\Gamma. \tag{6d}$$

At the symmetric equilibrium, the federal transfers policy satisfies the following first-order conditions:

$$\frac{\partial v}{\partial G_i} = \frac{\partial v}{\partial G_k} \quad \forall\, i,k \Rightarrow\ G_i = G_k \ \forall\, i,k, \tag{7}$$

$$\frac{\partial v}{\partial G_i} = 1 \quad \forall\, i. \tag{8}$$

The federal government allocates transfers to regions so as to (i) achieve horizontal equity, i.e. equalize the marginal utilities of current public expenditures in the federation, which boils down to identical current public expenditures in every region since households have the same preferences and (ii) ensure an optimal amount of current public expenditures in each region. To derive the best-reply of the federal government, $\{T_i^{gross}(\tau_1,\ldots,\tau_n, I_1,\ldots,I_n)\}_{i=1,\ldots,n}$ and $\Gamma^{gross}(\tau_1,\ldots,\tau_n,I_1,\ldots,I_n)$, to a change in a region i's tax rate τ_i and capital public expenditures I_i, we differentiate the condition[15] (8) w.r.t. T_i, T_{-i} and τ_i, I_i and we use the federal budget constraint[16]:

$$\frac{\partial T_i^{gross}}{\partial \tau_i} = -\frac{\partial \tau_i K_i}{\partial \tau_i} = -(1+\varepsilon_i)K_i < 0, \qquad \frac{\partial T_{-i}^{gross}}{\partial \tau_i} = -\frac{\partial \tau_{-i}K_{-i}}{\partial \tau_i} < 0, \tag{9a}$$

$$\frac{\partial T_i^{gross}}{\partial I_i} = 1 - \tau_i K_{I_i} - \tau_i K_{r_i}\frac{\partial \rho}{\partial I_i} > 0, \qquad \frac{\partial T_{-i}^{gross}}{\partial I_i} = -\tau_{-i}K_{r_{-i}}\frac{\partial \rho}{\partial I_i} > 0, \tag{9b}$$

$$\frac{\partial \Gamma^{gross}}{\partial \tau_i} = -\frac{1}{n}\left(\frac{\partial \tau_i K_i}{\partial \tau_i} + (n-1)\frac{\partial \tau_{-i}K_{-i}}{\partial \tau_i}\right) < 0, \tag{9c}$$

$$\frac{\partial \Gamma^{gross}}{\partial I_i} = \frac{1}{n}\left(1 - \tau_i K_{I_i} - \tau_i K_{r_i}\frac{\partial \rho}{\partial I_i} - (n-1)\tau_{-i}K_{r_{-i}}\frac{\partial \rho}{\partial I_i}\right) > 0. \tag{9d}$$

Note that the whole amount of ex-post transfers to region i is given by $dT_i^{gross} = \frac{\partial T_i^{gross}}{\partial \tau_i}d\tau_i + \frac{\partial T_i^{gross}}{\partial I_i}dI_i$. In addition, for $\frac{\partial T_i^{gross}}{\partial \tau_i} = \frac{\partial T_{-i}^{gross}}{\partial \tau_i} = \frac{\partial T_i^{gross}}{\partial I_i} = \frac{\partial T_{-i}^{gross}}{\partial I_i} = \frac{\partial \Gamma^{gross}}{\partial \tau_i} = \frac{\partial \Gamma^{gross}}{\partial I_i} = 0$, regional budgetary decisions would be those taken without decentralized leadership.

Achieving an optimal amount of current public expenditures in each region, $v' = 1\ \forall\, i$, implies that any reduction in a region i's tax rate must be compensated by both a raise in the region i's transfer and a raise in the other regions' transfers. Indeed, a reduction in the region i's tax rate τ_i lowers tax revenues and thus current public expenditures in region i $\left(-\frac{\partial \tau_i K_i}{\partial \tau_i} < 0\right)$ and also in the other regions $\left(-\frac{\partial \tau_{-i}K_{-i}}{\partial \tau_i} < 0\right)$, ceteris paribus, which no longer satisfies the condition (8), in the absence of federal ex-post intervention. As the capital locates where the net return of capital is the highest, a reduction in τ_i generates a capital inflow in the region i, at the expense of the other regions whose tax bases are reduced. But the resulting increase in region i's tax base, $-\varepsilon_i K_i > 0$, does not compensate the loss of tax revenues, $-K_i$, given that $(1+\varepsilon_i) > 0$. The federal government is obliged to react ex-post to these tax revenues losses by bailing out all the regions, including region i. Through a reduction in τ_i, region i is thus able to attract extra transfers for itself, and also for the other regions. These ex-post transfers are entirely financed by a raise in the lump-sum tax Γ, i.e. $\sum_{i=1}^{n} dT_i^{net} = n\,d\Gamma$.

Achieving an optimal amount of current public expenditures in each region also implies that the budgetary impact of any increase in a region i's capital public expenditures must be compensated by the federal government. The negative externality generated by an increase in I_i on the tax base of each region – which goes through the net return of capital ρ – always induces extra transfers from the top to all regions to entirely compensate the tax revenues losses. The sign of the variation of the transfer granted to region i is positive, as we have seen before.

4.2.2. With a net equalization scheme

We now assume that the federal government no longer chooses the size of the equalization pool but still determines the equalization rule, that is the allocation of transfers. In that setting, any additional transfer to one region must be financed by a reduction in transfers granted to the other regions. The problem for the federal government is the same as (6) except that the federal lump-sum tax is no longer endogenous:

$$\max_{\mathbf{T}} \sum_{i=1}^{n}[c_i + v(G_i)] \tag{10a}$$

$$\text{s.t. } c_i = \Pi_i(r_i, I_i) + \rho\widetilde{K} - \overline{T}, \tag{10b}$$

$$I_i + G_i = T_i + \tau_i K_i(r_i, I_i), \tag{10c}$$

$$\sum_{i=1}^{n} T_i = n\overline{T} \quad \text{with} \quad 0 \leqslant \overline{T} \leqslant \gamma. \tag{10d}$$

At the symmetric equilibrium, the federal government still cares for interregional equity, i.e. allocates transfers to regions so as to equalize the marginal utilities of current public expenditures in the federation:

$$\frac{\partial v}{\partial G_i} = \frac{\partial v}{\partial G_k} \qquad \forall\, i,k \Rightarrow G_i = G_k \ \forall\, i,k, \tag{11}$$

but nothing guarantees an optimal amount of current public expenditures. To derive the best-reply of the federal government,

[15] $v''\left[\frac{\partial \tau_i K_i}{\partial \tau_i}d\tau_i + \left(\tau_i K_{I_i} + \tau_i K_{r_i}\frac{\partial \rho}{\partial I_i}\right)dI_i + dT_i - dI_i\right] = 0$.

[16] The assumption $1 - \tau_i K_{I_i} - \tau_i K_{r_i}\frac{\partial \rho}{\partial I_i} \geqslant 0$ ensures that $\frac{\partial T_i^{gross}}{\partial I_i} > 0$.

$\{T_i^{net}(\tau_1,\dots\tau_n,\ I_1,\dots,I_n)\}_{i=1,\dots,n}$, to a change in a region i's tax rate τ_i and capital public expenditures I_i, we differentiate the condition (11) w.r.t. T_i, T_{-i} and $\tau_i, I_i, \tau_{-i}, I_{-i}$:

$$\begin{aligned}
&v''\left[\frac{\partial\tau_i K_i}{\partial\tau_i}d\tau_i+\sum_{-i}\tau_i\left(\frac{\partial K_i}{\partial\tau_{-i}}d\tau_{-i}+K_{r_i}\frac{\partial\rho}{\partial I_{-i}}dI_{-i}\right)\right.\\
&\quad\left.+\left(\tau_i K_{l_i}+\tau_i K_{r_i}\frac{\partial\rho}{\partial I_i}\right)dI_i+dT_i-dI_i\right]\\
&=v''\left[\frac{\partial\tau_j K_j}{\partial\tau_j}d\tau_j+\sum_{-j}\tau_j\left(\frac{\partial K_j}{\partial\tau_{-j}}d\tau_{-j}+K_{r_j}\frac{\partial\rho}{\partial I_{-j}}dI_{-j}\right)\right.\\
&\quad\left.+\left(\tau_j K_{l_j}+\tau_j K_{r_j}\frac{\partial\rho}{\partial I_j}\right)dI_j+dT_j-dI_j\right].
\end{aligned}$$

Summing across j, for $j\neq i$, and using the federal budget constraint $\sum_{i=1}^n T_i=n\overline{T}$ yields to:

$$\frac{\partial T_i^{net}}{\partial\tau_i}=-\frac{(n-1)}{n}\frac{\partial\tau_i K_i}{\partial\tau_i}+\frac{1}{n}\sum_{-i}\tau_{-i}\frac{\partial K_{-i}}{\partial\tau_i}<0, \tag{12a}$$

$$\frac{\partial T_{-i}^{net}}{\partial\tau_i}=\frac{1}{n}\frac{\partial\tau_i K_i}{\partial\tau_i}-\frac{1}{n}\tau_{-i}\frac{\partial K_{-i}}{\partial\tau_i}>0, \tag{12b}$$

$$\frac{\partial T_i^{net}}{\partial I_i}=\frac{(n-1)}{n}\left(1-\tau_i K_{l_i}-\tau_i K_{r_i}\frac{\partial\rho}{\partial I_i}\right)+\frac{1}{n}\sum_{-i}\tau_{-i}K_{r_{-i}}\frac{\partial\rho}{\partial I_i}, \tag{12c}$$

$$\frac{\partial T_{-i}^{net}}{\partial I_i}=-\frac{1}{n}\left(1-\tau_i K_{l_i}-\tau_i K_{r_i}\frac{\partial\rho}{\partial I_i}\right)-\frac{1}{n}\tau_{-i}K_{r_{-i}}\frac{\partial\rho}{\partial I_i}. \tag{12d}$$

The whole amount of ex-post transfers to region i is given by $dT_i^{net}=\frac{\partial T_i^{net}}{\partial\tau_i}d\tau_i+\frac{\partial T_i^{net}}{\partial I_i}dI_i$. Like Köthenbürger (2004), an increase in τ_i exerts two opposite effects on the federal transfer to region i: on the one hand, any increase in region i's tax revenues is captured by the federal government to be redistributed equally among all the regions; on the other hand, any capital outflow from region i, which increases other regions' tax revenues, is partially compensated by contributions made by the other regions, which ensures the equalization of marginal utilities ex-post. The global influence of a raise (resp. decrease) in region i's tax effort, combining these two effects, is a reduction (resp. an increase) in region i's federal transfer which benefits (resp. harms) the other regions. In a standard manner, the externalities linked to capital mobility across regions are perfectly internalized by the transfers scheme designed by the federal government.

But unlike Köthenbürger (2004), the federal transfers also react to the regional capital public expenditures. The federal government smooths the impact of a change in I_i across regions so as to equalize marginal utilities ex-post. An increase in capital public expenditures exerts two opposite effects on capital and thus on the transfer to region i. Firstly, an increase in I_i attracts more capital, that is, additional tax revenues in the region i which are captured by the federal government to be redistributed equally among all regions. Secondly, an increase in I_i generates negative externalities for all regions, via a rise in the interest rate, which are shared among regions. Without further assumptions, we cannot sign $\frac{\partial T_i^{net}}{\partial I_i}$ and $\frac{\partial T_{-i}^{net}}{\partial I_i}$.

The comparison between the best-reply function with a gross equalization scheme (9) and the best-reply function with a net equalization scheme (12) yields the following proposition:

Proposition 1. *The net equalization scheme, compared with the gross equalization scheme, always*

(i) *reduces ex-post transfers following a reduction in the regional tax rate,*

(ii) *reduces ex-post transfers following an increase in the regional capital public expenditures under the assumption* $1-\tau_i K_{l_i}-\tau_i K_{r_i}\frac{\partial\rho}{\partial I_i}>0$.

Proof. Directly from the comparison of (9) with (12). □

As a result, the net equalization scheme reduces ex-post intervention and thus the deviation from the optimum. The size of the net equalization pool, i.e. the amount of tax revenues $n\overline{T}$ initially devoted to equalization, does not affect the amount of ex-post transfers and the incentives at the margin:

$$\frac{\partial}{\partial\overline{T}}\left(\frac{\partial T_i^{net}}{\partial\tau_i}\right)=\frac{\partial}{\partial\overline{T}}\left(\frac{\partial T_{-i}^{net}}{\partial\tau_i}\right)=\frac{\partial}{\partial\overline{T}}\left(\frac{\partial T_i^{net}}{\partial I_i}\right)=\frac{\partial}{\partial\overline{T}}\left(\frac{\partial T_{-i}^{net}}{\partial I_i}\right)=0.$$

The mere fact that ex-post transfers are self-financed by regions, i.e. $\sum_{i=1}^n dT_i^{net}=0$, reduces the amount of ex-post transfers each region can expect from a change in its policy, compared with a gross equalization scheme.

Note that when the number of regions n is large, the additional lump-sum tax levied on each citizen to finance ex-post transfers with a gross equalization scheme tends to zero and the transfers tend to have the same extent whichever the equalization scheme. The revenue sharing system (via a rise in the federal lump-sum tax with a gross equalization scheme or via a rise in the contribution of each region with a net equalization scheme) allocates only a negligible fraction of the social marginal effect to each region.

4.3. The regional government's problem

We now analyze how a gross versus net equalization scheme affects regional budgetary choices.

4.3.1. With a gross equalization scheme

Each region i's government acts as a Stackelberg leader vis-à-vis the federal government. It maximizes the utility of the representative household located in its territory taking into account the best-reply of the federal government, $T_i^{gross}(\tau_1,\dots\tau_n,\ I_1,\dots,I_n)$ and $\Gamma^{gross}(\tau_1,\dots,\tau_n,I_1,\dots,I_n)$:

$$\begin{aligned}
&\max_{\tau_i,I_i}\ c_i+v(G_i),\\
&\text{s.t. } c_i=\Pi_i(r_i,I_i)+\rho\widetilde{K}-\Gamma^{gross},\\
&\quad I_i+G_i=T_i^{gross}+\tau_i K_i(r_i,I_i),\\
&\quad\sum_{i=1}^n T_i^{gross}=n\Gamma^{gross},
\end{aligned}$$

which yields to the following first-order conditions[17]:

$$\Pi_{r_i}\frac{\partial r_i}{\partial\tau_i}+\frac{\partial\rho}{\partial\tau_i}\widetilde{K}-\frac{\partial\Gamma^{gross}}{\partial\tau_i}+v'\left[\frac{\partial\tau_i K_i}{\partial\tau_i}+\frac{\partial T_i^{gross}}{\partial\tau_i}\right]=\Pi_{r_i}-\frac{\partial\Gamma^{gross}}{\partial\tau_i}<0, \tag{13}$$

$$\Pi_{l_i}+\Pi_{r_i}\frac{\partial\rho}{\partial I_i}+\frac{\partial\rho}{\partial I_i}\widetilde{K}-\frac{\partial\Gamma^{gross}}{\partial I_i}+v'\left[-1+\tau_i K_{l_i}+\tau_i K_{r_i}\frac{\partial\rho}{\partial I_i}+\frac{\partial T_i^{gross}}{\partial I_i}\right]=0.$$

The terms in square brackets in the FOCs are null as the federal government entirely compensates, via its transfers policy, the impact of a variation in the regional taxation or in the capital public expenditures, to achieve an optimal amount of current public expenditures. Note that from (13) the regional tax rate is equal

[17] Using the capital market clearing condition $\sum_{i=1}^n K_i(r_i,I_i)=n\widetilde{K}$, $\Pi_{l_i}=F_{l_i}$, $\Pi_{r_i}=-K_i$, $\frac{\partial\rho}{\partial I_i}=-\frac{K_{l_i}}{nK_{r_i}}$, and $\frac{\partial\rho}{\partial\tau_i}=-\frac{1}{n}$, at the symmetric equilibrium, we obtain: $\Pi_{r_i}\frac{\partial\rho}{\partial I_i}+\frac{\partial\rho}{\partial I_i}\widetilde{K}=-K_i\frac{\partial\rho}{\partial I_i}+\frac{\partial\rho}{\partial I_i}\widetilde{K}=0$; $\frac{\partial\Gamma^{gross}}{\partial I_i}=\frac{1}{n}(1-\tau_i K_{l_i}-\tau_i K_{r_i}\frac{\partial\rho}{\partial I_i}-(n-1)\tau_{-i}K_{r_{-i}}\frac{\partial\rho}{\partial I_i})=\frac{1}{n}$ $(1-\tau_i K_{l_i}+n\tau_i K_{r_i}\frac{K_{l_i}}{nK_{r_i}})=\frac{1}{n}$; $\frac{\partial\Gamma^{gross}}{\partial\tau_i}=-\frac{1}{n}\left(\frac{\partial\tau_i K_i}{\partial\tau_i}+(n-1)\tau_{-i}\frac{\partial K_{-i}}{\partial\tau_i}\right)=-\frac{1}{n}(K_i+\tau_i K_{r_i}\frac{(n-1)}{n}-\frac{(n-1)}{n}\tau_{-i}K_{r_{-i}})=-\frac{1}{n}K_i$.

to zero at the equilibrium because collecting one unit of tax revenue always reduces the utility of the representative consumer ($\frac{\partial U}{\partial \tau_i} = \frac{\partial c_i}{\partial \tau_i} < 0$ given that $\frac{\partial G_i}{\partial \tau_i} = 0$). Regional public expenditures are thus entirely financed by federal transfers.

At the symmetric equilibrium, the condition w.r.t. I_i boils down to:

$$\Pi_{I_i} = \frac{\partial \Gamma^{gross}}{\partial I_i} \Longleftrightarrow F_{I_i} = \frac{1}{n} < 1. \tag{14}$$

The amount of capital public expenditures at the equilibrium equalizes the marginal profit (Π_{I_i}) to the marginal increase in the federal lump-sum tax ($\frac{\partial \Gamma^{gross}}{\partial I_i}$). This condition, which boils down to $F_{I_i} = \frac{1}{n} < 1$, implies that too much capital public expenditures are financed by each region relative to the first-best optimum, i.e. the public input is overprovided. Regions are incited to finance more capital public expenditures since they know that they will bear only a fraction $\frac{1}{n}$ of the cost, thanks to ex-post federal transfers financed through an increase in Γ. On the one hand, regions anticipate – despite no regional tax revenues – that the amount of current public expenditures will be optimal, since the federal government bails out the regions to compensate entirely for the loss of tax revenue and to ensure $v'(G_i) = 1\ \forall i$. On the other hand, the cost of one additional dollar of capital public expenditures will be financed by a rise in the federal lump-sum tax which implies that the cost for the representative citizen in region i only amounts to $\frac{1}{n}$.

These results can be summarized by the following proposition:

Proposition 2. *The equilibrium with a gross equalization scheme is characterized by an optimal amount of current public expenditures ($v'(G_i) = 1$), and an upward distortion of the amount of capital public expenditures w.r.t. the optimum ($F_{I_i} = \frac{1}{n} < 1$).*

4.3.2. With a net equalization scheme

The regional government in i maximizes the utility of the representative household located in its territory taking into account the best-reply function of the federal government $T_i^{net}(\tau_1, \ldots \tau_n, I_1, \ldots, I_n)$:

$$\begin{aligned} &\max_{I_i, \tau_i} c_i + v(G_i), \\ &\text{s.t. } c_i = \Pi_i(r_i, I_i) + \rho\widetilde{K} - \overline{\Gamma}, \\ &\quad I_i + G_i = T_i^{net} + \tau_i K_i(r_i, I_i), \\ &\quad \sum_{i=1}^{n} T_i^{net} = n\overline{\Gamma} \quad \text{with} \quad 0 \leqslant \overline{\Gamma} \leqslant \gamma, \end{aligned}$$

which yields the following first-order conditions:

$$\Pi_{r_i}\frac{\partial r_i}{\partial \tau_i} + \frac{\partial \rho}{\partial \tau_i}\widetilde{K} + v'\left[\frac{\partial \tau_i K_i}{\partial \tau_i} + \frac{\partial T_i^{net}}{\partial \tau_i}\right] = 0,$$

$$\Pi_{I_i} + \Pi_{r_i}\frac{\partial \rho}{\partial I_i} + \frac{\partial \rho}{\partial I_i}\widetilde{K} + v'\left[-1 + \tau_i K_{I_i} + \tau_i K_{r_i}\frac{\partial \rho}{\partial I_i} + \frac{\partial T_i^{net}}{\partial I_i}\right] = 0.$$

At the symmetric equilibrium, using $\Pi_{r_i} = -K_i$ and the capital market clearing condition $\sum_{i=1}^{n} K_i(r_i, I_i) = n\widetilde{K}$, replacing $\frac{\partial T_i^{net}}{\partial I_i}$ and $\frac{\partial T_i^{net}}{\partial \tau_i}$ by their values, the conditions boil down to[18]:

$$v'(G_i) = n > 1, \tag{15}$$

$$\Pi_{I_i} = v'(G_i)\cdot\frac{1}{n} \Longleftrightarrow F_{I_i} = 1. \tag{16}$$

In line with Köthenbürger (2004), the amount of current public expenditures is distorted downwards, i.e. the public good is underprovided with respect to the first-best solution. Increasing the tax rate by one unit reduces private good consumption by K_i but increases public funds devoted to current public expenditures only by $\frac{1}{n}K_i$ due to revenue sharing.[19] Note that the fact that the federal government is no longer able to change the size of the equalization pool $n\overline{\Gamma}$ now forces the regional government to set a positive tax rate.

In addition, the amount of capital public expenditures chosen at the symmetric equilibrium by each region turns out to be optimal, as implied by the condition $F_{I_i} = 1$. Increasing capital public expenditures by one unit increases profit by $F_{I_i}(= \Pi_{I_i})$ but reduces public funds devoted to current public expenditures by $\frac{1}{n}$ due to revenue sharing, since the federal government cannot maintain an optimal amount of current public expenditures. Actually, the two distortions due to revenue sharing, that is $v'(G_i) = n$ and $\frac{\partial G_i}{\partial I_i} = \frac{1}{n}$, offset as $v'(G_i)\frac{\partial G_i}{\partial I_i} = 1$, which ensures an optimal amount of capital public expenditures.

Compared to the absence of decentralized leadership, the net equalization scheme with decentralized leadership restores an optimal choice of capital public expenditures as the transfer scheme designed by the federal government perfectly internalizes the externalities linked to the fiscal competition.[20]

These results can be summarized by the following proposition:

Proposition 3. *The equilibrium with a net equalization scheme is characterized by a downward distortion of the amount of current public expenditures w.r.t. the optimum ($v'(G_i) = n > 1$), and an optimal provision of the amount of capital public expenditures ($F_{I_i} = 1$).*

The comparison between (14) and (16) emphasizes that more capital public expenditures are financed when the federal government implements a gross equalization scheme than when it implements a net equalization scheme. Moreover, the comparison between (8), (13) and (15) shows that the regional government sets a lower tax rate and more current public expenditures are financed when the federal government implements a gross equalization scheme than when it implements a net equalization scheme.

Interestingly, the fact that the two schemes differentiate by the distortive nature of tax revenues that are raised to finance ex-post transfers – i.e. the gross equalization scheme involves lump-sum taxes whereas the net equalization scheme involves distortionary taxes – plays no role because ex-post transfers perfectly internalize tax externalities. Indeed, suppose that each region i levies a non-distortive lump-sum tax φ_i on immobile households instead of a distortive tax τ_i on mobile capital,[21] so that ex-post transfers are financed by non-distortive taxation under both equalization schemes. The best-reply of the federal government under a gross equalization scheme (resp. under a net equalization scheme) becomes $\frac{\partial T_i^{gross}}{\partial \varphi_i} = -1, \frac{\partial T_{-i}^{gross}}{\partial \varphi_i} = 0$, $\frac{\partial T_i^{gross}}{\partial I_i} = 1, \frac{\partial T_{-i}^{gross}}{\partial I_i} = 0$, $\frac{\partial T^{gross}}{\partial \varphi_i} = -\frac{1}{n}, \frac{\partial T^{gross}}{\partial I_i} = \frac{1}{n}$ (resp. $\frac{\partial T_i^{net}}{\partial \varphi_i} = -\frac{(n-1)}{n}, \frac{\partial T_{-i}^{net}}{\partial \varphi_i} = \frac{1}{n}$, $\frac{\partial T_i^{net}}{\partial I_i} = \frac{(n-1)}{n}, \frac{\partial T_{-i}^{net}}{\partial I_i} = -\frac{1}{n}$). The maximization of the regional utility w.r.t. φ_i and I_i, taking into account the best-reply of the federal government, yields – after simplification – exactly the same FOCs as the ones with distortive regional taxation, i.e. $F_{I_i} = \frac{1}{n}$ and $v'(G_i) = 1$ under a gross equalization scheme, and $F_{I_i} = 1$ and $v'(G_i) = n$ under a net equalization scheme.

In addition, note that the size of the net equalization pool, $n\overline{\Gamma}$, has no impact on the provision of both current public expenditures and capital public expenditures. Only the regional tax pressure is modified. The lower $\overline{\Gamma}$, the lower T_i^{net}, which forces the regional

[18] Using $\frac{\partial r_i}{\partial \tau_i}$ and $\frac{\partial \rho}{\partial \tau_i}$, the expression $\frac{1}{n}\frac{\partial \tau_i K_i}{\partial \tau_i} + \frac{(n-1)}{n}\tau_{-i}\frac{\partial K_{-i}}{\partial \tau_i}$ simplifies to $\frac{1}{n}K_i$. By using the fact that $-1 + \tau_i K_{I_i} + \tau_i K_{r_i}\frac{\partial \rho}{\partial I_i} + \frac{\partial T_i^{net}}{\partial I_i}$ simplifies into $-1 + \frac{n-1}{n}$, and $v'(G_i) = n$.

[19] $\Pi_{r_i}\frac{\partial r_i}{\partial \tau_i} + \frac{\partial \rho}{\partial \tau_i}\widetilde{K} = -K_i$ at the capital market equilibrium and $\frac{\partial \tau_i K_i}{\partial \tau_i} + \frac{\partial T_i^{net}}{\partial \tau_i} = -\frac{1}{n}K_i$.

[20] The role of federal transfers in internalizing externalities linked to competition among regions via their capital public expenditures can be seen by comparing the FOC w.r.t. I_i in the absence of decentralized leadership, i.e. $F_{I_i} = v'\left[1 - \tau_i K_{I_i} - \tau_i K_{r_i}\frac{\partial \rho}{\partial I_i}\right]$ and the FOC w.r.t. I_i with decentralized leadership, i.e. $F_{I_i} = v'\left[1 - \tau_i K_{I_i} - \tau_i K_{r_i}\frac{\partial \rho}{\partial I_i} + \frac{\partial T_i^{net}}{\partial I_i}\right] = 1$.

[21] Budget constraints change as follows: $c_i = \Pi_i(r_i, I_i) + \rho\widetilde{K} - \Gamma - \varphi_i$ and $I_i + G_i = T_i + \varphi_i$.

government to increase its tax rate in order to provide current and capital public expenditures such as defined by (15) and (16).

To sum up, too much is spent in capital public expenditures and too little taxes are levied by the region with a gross equalization scheme. This strategic behaviour is driven by the fact that it aims at extracting more ex-post transfers from the top. The net equalization scheme must be favored over the gross equalization scheme if the federal government wants to instill more fiscal discipline at the regional level. However, the respective impact of both equalization schemes on the welfare is uncertain at first sight.

4.4. The welfare analysis

4.4.1. The composition of public expenditures

In line with Keen and Marchand (1997),[22] we first analyze how welfare would change following a rebalancing of public expenditures. Starting at the non-cooperative equilibrium, we suppose that regions coordinate to modify the mix of their expenditures ceteris paribus, i.e. keeping unchanged federal and regional tax rates, the allocation of transfers and the location of capital. They decide to modify the respective share of both current and capital public expenditures in the total amount of public expenditures, so that $dI_i + dG_i = 0 \ \forall\, i$. The effect on the welfare of their representative citizen of rebalancing public expenditures is:

$$dU(c_i, G_i) = dc_i + v'(G_i)dG_i,$$

where $dc_i = \left(\Pi_{I_i} + \Pi_{r_i}\frac{\partial \rho}{\partial I_i} + \frac{\partial \rho}{\partial I_i}\widetilde{K}\right)dI_i = F_{I_i}dI_i$ at the symmetric equilibrium. Using $dI_i = -dG_i$, we obtain:

$$dU(c_i, G_i) = (F_{I_i} - v'(G_i))dI_i \quad \text{and} \quad dU_i = (-F_{I_i} + v'(G_i))dG_i. \qquad (17)$$

Substituting the first-order conditions, which characterize the non-cooperative equilibrium under a gross equalization scheme, i.e. $F_{I_i^{gross}} = \frac{1}{n}$ and $v'(G_i^{gross}) = 1$, into (17) yields:

$$\frac{dU_i^{gross}}{dI_i^{gross}} = \frac{1}{n} - 1 < 0 \quad \text{and} \quad \frac{dU_i^{gross}}{dG_i^{gross}} = -\frac{1}{n} + 1 > 0.$$

Similarly, substituting the first-order conditions, which characterize the non-cooperative equilibrium under a net equalization scheme, i.e. $F_{I_i^{net}} = 1$ and $v'(G_i^{net}) = n$, into (17) yields:

$$\frac{dU_i^{net}}{dI_i^{net}} = 1 - n < 0 \quad \text{and} \quad \frac{dU_i^{net}}{dG_i^{net}} = -1 + n > 0.$$

Whichever the equalization scheme, welfare would be unambiguously improved by a coordinated reduction in capital public expenditures and a corresponding increase in current public expenditures. The loss of welfare due to the inefficient mix in public expenditures is even higher when a net equalization scheme is implemented, since $\frac{dU_i^{net}}{dG_i^{net}} > \frac{dU_i^{gross}}{dG_i^{gross}}$ and $\left|\frac{dU_i^{net}}{dI_i^{net}}\right| > \left|\frac{dU_i^{gross}}{dI_i^{gross}}\right|$. These results can be summarized by the following proposition:

Proposition 4. *At the symmetric non-cooperative equilibrium, welfare would be increased ceteris paribus if regions coordinate to decrease capital public expenditures and proportionately increase current public expenditures. The potential welfare gain from this shift is higher under a net equalization scheme.*

We therefore extend the result derived in the one-tier framework by Keen and Marchand (1997) to our two-tier federation with decentralized leadership. Although we show that the welfare gain from substituting one dollar of capital public expenditures by one dollar of current public expenditures is higher under a net equalization scheme, we do not know whether one equalization scheme is welfare superior to the other. This is the purpose of the next part.

4.4.2. The welfare comparison between gross and net equalization schemes

Given the symmetry of our model, the welfare comparison between equalization schemes relies on the comparison between the utility a given household derives respectively from the gross equalization scheme, U_i^{gross}, and from the net equalization scheme, U_i^{net}. Hence, the net equalization scheme is welfare-superior if and only if $U_i^{net} - U_i^{gross} > 0$. At the symmetric equilibrium, expressions of both U_i^{gross} and U_i^{net} can be simplified[23] into $F(\widetilde{K}, I_i) - I_i - G_i + v(G_i)$.

As shown before, a gross equalization scheme leads to an upward distortion of capital public expenditures, i.e. I_i^{gross} satisfies $F_{I_i} = \frac{1}{n}$, and an optimal amount of current public expenditures, i.e. $v'(G_i^{gross}) = 1$. As for a net equalization scheme, it leads to an optimal amount of capital public expenditures, i.e. I_i^{net} satisfies $F_{I_i} = 1$, and a downward distortion of current public expenditures, i.e. $v'(G_i^{net}) = n$.

For the particular case $n = 1$, welfare is obviously equivalent under both equalization schemes, $U_i^{gross} = U_i^{net}$. The single region perfectly internalizes the impact of its budgetary choices and provides an optimal amount of capital and current public expenditures.

For $n > 1$, it is far more difficult to compare the welfare levels under the two equalization schemes. As the amount of capital public expenditures is optimal only under the net equalization scheme, it can be inferred that $F(\widetilde{K}, I_i^{net}) - F(\widetilde{K}, I_i^{gross}) - (I_i^{net} - I_i^{gross})$ is always positive. Similarly, as the amount of current public expenditures is optimal only under the gross equalization scheme, it can be inferred that $-(G_i^{net} - G_i^{gross}) + v(G_i^{net}) - v(G_i^{gross})$ is always negative. But we are not able to state which distortion affects welfare more, without the specification of the functions $F(\cdot)$ and $v(\cdot)$. We only know that U_i^{net} always dominates U_i^{gross} if and only if $(F_{I_i} - 1)\frac{dI_i^{gross}}{dn} < (v'(G_i) - 1)\frac{dG_i^{net}}{dn}$, which ensures that $U_i^{net} - U_i^{gross}$ is an increasing function of n, given that $U_i^{gross} = U_i^{net}$ for $n = 1$.

Under a gross equalization scheme, a rise in n increases I_i^{gross} upward distortion (without altering G_i^{gross}) as the marginal cost that the representative citizen located in region i bears, i.e. $\frac{\partial I^{gross}}{\partial I_i} (= \frac{1}{n})$, shrinks with the number of regions financing additional capital public expenditures undertaken by region i. Under a net equalization scheme, a rise in n increases G_i^{net} downward distortion (without altering I_i^{net}) as the marginal cost of public funds n becomes higher, due to revenue sharing. In order to ensure that $U_i^{net} - U_i^{gross}$ is an increasing function of n, welfare variation due to higher G_i^{net} downward distortion must be higher than welfare variation due to higher I_i^{net} upward distortion.

In order to obtain analytical results, we therefore specify the key factors determining which equalization scheme is welfare-superior, i.e. (i) the production technology, $F(\widetilde{K}, I_i)$ and (ii) the preferences for current public expenditures, $v(G_i)$. Consider a standard Cobb–Douglas production technology, $F(K_i, I_i) = AK_i^{\alpha} I_i^{\beta}$, where the parameters $A > 0$, $\alpha \in]0,1[$ and $\beta \in]0,1[$ respectively measure the total factor productivity, the output elasticity of capital and the output elasticity of capital public expenditures. Preferences for current public expenditures are assumed to be logarithmic, $v(G_i) = \ln(G_i)$.

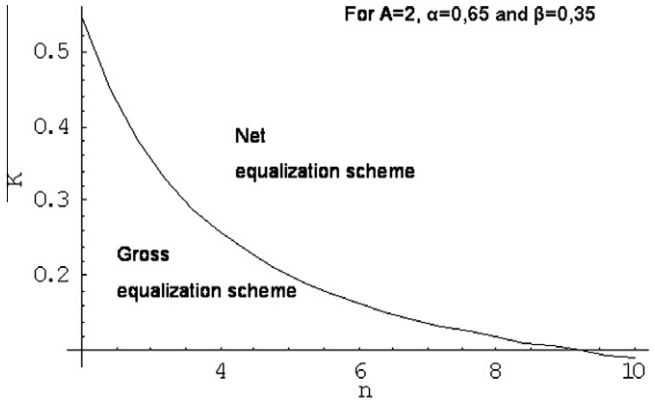
[22] Our framework is close to their special case without taxes on elastically supplied labour and pure profits, i.e. that of Zodrow and Mieszkowski (1986).

[23] Symmetry on the capital market implies $K_i = \widetilde{K}$, and thus $-r_i K_i + \rho\widetilde{K} = -\tau_i \widetilde{K}$. Using $\tau_i^{gross} = 0, I^{gross} = I_i^{gross} + G_i^{gross}$, $U_i^{gross} = F(K_i, I_i^{gross}) - r_i K_i + \rho\widetilde{K} - \Gamma + v(G_i^{gross})$ boils down to $F(\widetilde{K}, I_i^{gross}) - I_i^{gross} - G_i^{gross} + v(G_i^{gross})$. Using $\tau_i^{net} = \frac{I_i^{net} + G_i^{net} - T_i^{net}}{K_i}$, $T_i^{net} = \overline{\Gamma}$, $U_i^{net} = F(K_i, I_i^{net}) - r_i K_i + \rho\widetilde{K} - \overline{\Gamma} + v(G_i^{net})$ boils down to $F(\widetilde{K}, I_i^{net}) - I_i^{net} - G_i^{net} + v(G_i^{net})$.

Under a gross equalization scheme, the representative citizen derives a utility $U_i^{gross} = A^{\frac{1}{1-\beta}}\widetilde{K}^{\frac{\alpha}{1-\beta}}((n\beta)^{\frac{\beta}{1-\beta}} - (n\beta)^{\frac{1}{1-\beta}}) - 1 + \ln(1)$, from the regional and federal budgetary decisions taken at the equilibrium, i.e. $I_i^{gross} = (n\beta A\widetilde{K}^{\alpha})^{\frac{1}{1-\beta}}$, $G_i^{gross} = 1, \tau_i^{gross} = 0$ and $\Gamma^{gross} = I_i^{gross} + G_i^{gross}$. Under a net equalization scheme, $I_i^{net} = (\beta A\widetilde{K}^{\alpha})^{\frac{1}{1-\beta}} < I_i^{gross}$, $G_i^{net} = \frac{1}{n} < G_i^{gross}$, $\tau_i^{net} = \frac{I_i^{net}+G_i^{net}-\overline{\Gamma}}{\widetilde{K}}$, which gives a utility $U_i^{net} = A^{\frac{1}{1-\beta}}\widetilde{K}^{\frac{\alpha}{1-\beta}}(\beta^{\frac{\beta}{1-\beta}} - \beta^{\frac{1}{1-\beta}}) - \frac{1}{n} + \ln(\frac{1}{n})$ to the representative citizen. Note that the extent of $\overline{\Gamma}$ has no impact on the welfare.

Comparative statics confirm our expectation that the welfare gap $U_i^{net} - U_i^{gross}$ increases with respect to A, α and $\widetilde{K}$. In addition, we show that for a number of regions $n > \frac{(1-\beta)^{1-\beta}}{K^{\alpha}(A\beta)}$, $U_i^{net} - U_i^{gross}$ is an increasing function of n, which, given $U_i^{gross} = U_i^{net}$ for $n = 1$, ensures that the net equalization scheme is always welfare-superior.

The figure below depicts combinations of n and K for which the net equalization scheme is welfare-superior or welfare-inferior compared to the gross equalization scheme, for empirically consistent values of A, α and β, i.e. $A = 2$, $\alpha = 0{,}65$ and $\beta = 0{,}35$.



Incentives to use a net equalization scheme are thus strengthened for sufficiently high values of n.

5. Conclusion

In a decentralized leadership framework, this paper shows that a net equalization scheme favors fiscal discipline in comparison to a gross equalization scheme since it limits the competition among regions to attract ex-post transfers. Strategic regional behaviour, in terms of the composition of public expenditures, clearly changes depending on which equalization scheme at work. The gross equalization scheme leads to an optimal amount of current public expenditures and an upward distortion of capital public expenditures, which are induced by the expectations of high ex-post transfers financed by the federal government. With the net equalization scheme, the amount of current public expenditures becomes downwardly distorted due to the absence of ex-post federal tax revenues, but the optimal level of capital public expenditures is restored. Furthermore, we show that under both equalization schemes, and a fortiori under a net equalization scheme, welfare would be improved by a coordinated shift away from capital public expenditures towards current public expenditures. Finally, the welfare analysis reveals that the incentive to use a net equalization scheme is greater for countries with a large number of regions.

Although our paper is stylized, empirical testing may be conducted from our model using different episodes of fiscal equalization reforms across OECD countries. One way would be to run "natural experiments" in order to test whether changes in fiscal equalization schemes, which have been undertaken over the two last decades in most OECD countries, may have an impact on both fiscal discipline and the pattern of public expenditures at the regional level. However, setting up a data set which includes more or less all the OECD countries seems to be out of reach given that equalization arrangements are "tremendously country specific" (Blöchliger et al., 2007). In addition, equalization reforms are mostly embedded in a more general package of reforms, which includes the way fiscal responsibilities are divided between different levels of government. One way to circumvent such a problem is to use a country as a laboratory. Switzerland could provide a good field experiment. We have already observed that the growing amount of funds dedicated to fiscal equalization within the Swiss cantons since the 1990s has induced a significant increase in capital expenditures in comparison with the period before the reform. It seems that sub-national governments have taken advantage of extra equalizing funds to increase capital expenditures instead of current expenditures.

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