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Doctoral Thesis

BANK RISK APPETITE IN A WORLD OF COCOS

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"Fatti non foste a viver come bruti, ma per seguir virtute e canoscenza"

Dante Alighieri

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Contents

to my children

Introduction

In my PhD thesis I focus on bank capital structure, risk appetite and hybrid financing instruments, i.e. Contingent Convertible bonds (CoCos). I assess optimal bank financing and investment behaviour shaping its risk appetite. I conduct both a theoretical analysis with empirical application on the financing side and an empirical analysis on the investment one. The main goal of our research is to understand bank's risk appetite in presence of growth opportunities and a financing structure including CoCo bonds, hybrid capital securities that absorb losses when the capital of the issuing bank falls below a certain threshold. We conduct our analysis in two steps, assessing the bank risk appetite in presence of franchise value first in a context of standard financing, second introducing CoCo bonds. The thesis consists of three chapters. Chapter 1, "How to Shape Risk Appetite in Presence of Franchise Value?"(with G. Barone-Adesi), Chapter 2, "Optimal Bank Risk Appetite in a World of CoCos"(with G. Barone-Adesi) and Chapter 3 "CoCo Bonds and Write Down bonds impact on Banks'Risk Appetite and Investment Policy"(with T. Nefedova, G. Pratobevera and A. Ruzza).

Chapter 1: How to shape risk appetite in presence of franchise value? (with Giovanni Barone-Adesi)

The first paper is a joint work with Giovanni Barone-Adesi. We investigate the shape of the risk appetite of our bank and the role played by the monetary policy in framing it. Bank objective function and its risk appetite are determined by the interplay of the default option and the down-and-out call (DOC) option, pricing the franchise value, i.e. the net present value of non-observable bank's growth opportunities. We define the objective function as the ratio between the sum of the two options' prices and the market value of the tangible assets. Our major contribution consists in assessing risk appetite in three dimensions, allowing also the monetary policy to play a role on risk appetite and to work jointly with the bank manager in the optimization of the objective function. We test our optimizations on a sample of 1436 banks, listed in the US, over 1980-2014. We find that the optimal risk-free rate is higher with respect to the existing one in the last period. The objective function is magnified for lower values of leverage, which is straightforward given our specifications and optimal volatility should stay low in order not to erode the franchise value. The monetary policy maker should play a role for an effective risk appetite optimization. We show that regulators should tune

their recommendations depending on the targeted cluster, since the driver of risk appetite alternates between the two options depending on the cluster and on the underlying variable considered, given the other two. Furthermore, introducing the franchise value in the specification of risk appetite, we propose an incentive for the manager to adopt a policy long-term oriented. There is still ample room for the regulator to find the proper instruments in order to boost banks growth on one side, and consequently help economic growth, and to prevent them undertaking excessive risks on the other side.
Our paper is based on the seminal works by Black and Scholes (1973) and Merton (1973a), where the liabilities of a company are seen as an European option written on the assets of a firm. The endogenization of the default threshold, proposed by Leland and Toft (1996), provides alone not a clear improvement with respect to the standard Merton model, unless a jump component is introduced, as in Leland (2006). Our study is more related to Brockman and Turtle (2003), who introduce in equity path dependency, i.e. equity can be knocked out whenever a legally binding barrier is breached. Hugonnier and Morellec (2017) propose a dynamic model of banking assessing the impact of the main instruments in Basel III. On the other side, leverage requirements decrease default risk and increase growth opportunities of the bank, on the long-run, which is partly in line with our findings. Additionally, raising equity requirements make the loss to be borne by shareholders and the distance to default increases (see e.g. Admati and Hellwig (2013)). The model we propose relies on regulatory principles: Basel III indicates a bank is insolvent if the common equity tier one (from now on CET1) is below 4.5%. It is true that in some countries banks, that would be declared insolvent for Basel III, still run their assets. Thus, a possible extension to this model would consider the interplay between an exogenous default barrier set by the regulator and the endogenous one chosen by the bank, highlighting an important weakness in monitoring by the regulator.

Chapter 2: Optimal Bank Risk Appetite in a World of CoCos. (with Giovanni Barone-Adesi

In the post- Lehman Brothers failure, governments announced the end of the *too big to fail*. In this context, issuing loss absorbing instruments has gained increasing popularity. Between 2009 and 2015, banks issued more than 380 billion of CoCos [1]. Regulation plays a crucial role in determining CoCos'issuance. Under Basel III, CoCo bonds are eligible as either Additional Tier 1 (AT1) or Tier 2 (T2), which are types of capital apparently preferred by banks with

[1] Data from Moody's Investors Service, Moody's Quarterly Rated and Tracked CoCo Monitor Database-Year End 2015

respect to equity to accomplish regulatory requirements, given that they are cheaper and less dilutive than issuing equity. Their introduction into the financing structure of our banks is relevant from a regulation and risk management point of view.

In the second paper, prepared in collaboration with Giovanni Barone-Adesi, we introduce CoCo bonds in the financing structure of our bank and see how the shape of the bank risk appetite changes. Some of the characteristics of CoCo bonds are key in understanding the dynamics of risk appetite. In our model, the manager acts in order to accomplish regulatory requirements. Focusing on capital requirements, Basel III rule requires banks to fund themselves with at least 4.5% of common equity of risk-weighted assets (RWAs). The regulator allows for an extra 1.5% of Additional Tier 1 (AT1) that together with the CET1 concurs to compose the minimum level of 6% of Tier 1 capital over RWAs. Hence, in order to be compliant with this ratio, the manager has discretionary power over two variables: CoCos issuance and assets' volatility. On one side, issuing CoCos, which are eligible for AT1, the manager increases the numerator of the ratio, enlarging the Tier 1. On the other side, the manager might decrease the RWAs. RWAs are a weighted sum of banks' assets, thus we refer to this figure as the total assets' volatility. Through this capital requirement ratio, the regulator provides an incentive for decreasing assets' volatility. Optimizing the level of risk appetite, the manager should focus on the maximization of the bank objective function, which is given by the sum of the two options (default put option and down-and-out call option). In this model, we have an additional optimization variable, with respect to the basic model outlined in the first paper: the proportion of CoCos to issue with respect to the total amount of debt. Hence, the manager has discretionary power over the level of leverage, the proportion of CoCos to issue and the assets' and franchise value's volatility. The last optimizing variable is the policy rate: this is a relevant variable to be assessed in this context. Nowadays it is near zero or negative, thus it pushes the bank to substitute assets into more risky ones, pretending to get higher returns. In this framework, we argue that monetary policy makers drive the banks assets' volatility to higher levels, on the contrary the regulators attempt to mitigate this through the capital requirement ratio's incentive to decrease RWAs. Which one of the two incentives has the greater impact over our objective function? For higher levels of volatility, the default put option is magnified and the down-and-out call (DOC) option is more likely to expire worthless because of the higher probability of breaching the barrier. Vice versa, for lower levels of volatility, this probability is smaller, increasing the DOC option value even more thanks to the CoCos financing that are widening the distance to the barrier. Nevertheless, we consider also the case where their cushion function is weakened because of the consequent decrease in the market value of equity given that the market might discount

the CoCos' conversion as a signal of a forthcoming default. Our contribution to the existent literature, is again to assess risk appetite in a multi-dimensional perspective and to account for differences among banks' clusters which are even more relevant in a world with CoCos. Furthermore, we address the pros and cons of the regulation in force, based on a ratio giving interesting incentives and a monetary policy allowing for negative interest rates.
Attaoui and Poncet (2015) develop the model showing that credit spread on straight debt is lower if the firm has WD bonds in its financing structure, given the cushion function of the WDs with respect to the straight debt (senior). CoCos are nearer to equity because in some states of the world they are not debt. Chen et al. (2013) show that replacing some straight debt with CoCos lowers the endogenous default barrier and therefore increases the firm's ability to mitigate a loss in asset value. A natural direction for future research is to consider the impact of wealth transfer among different categories of stakeholders, which should be relevant for governments. Roy and El-Herraoui (2016) demonstrate the complexity of designing a fair and effective bail-in regime. The regulator is mainly confronted with the choice of implementing or not the wealth transfer. If it chooses to do so, it faces the risk of requests for compensation and arbitrage behavior in financial markets.

Chapter 3: CoCo Bonds and Write Down bonds impact on Banks' Risk Appetite and Investment Policy (with T. Nefedova, G. Pratobevera and A. Ruzza)
Theoretical literature has widely assessed hybrid capital, on the contrary there is a small empirical literature, given the scarcity data. Scepticism around CoCos come from their short track record, as they were introduced only in 2009 in the banking industry, making their performance not yet tested during bad times. In the paper, written jointly with Tamara Nefedova, Giuseppe Pratobevera and Alessio Ruzza, the main goal is to assess empirically banks' risk appetite and investment policy when their financing structure includes contingent convertible bonds (CoCos) and (or) write-down bonds (WDs), hybrid capital securities that absorb losses when the capital of the issuing bank falls below a certain threshold. The main difference between the two financing instruments is that CoCos convert to equity, while WDs convert to zero. We aim to study the impact on bank risk appetite of issuing WDs/CoCos, and what happens to the bank investment policy. To the best of our knowledge, this is the first attempt to address these questions empirically. From 2009 to 2014, about half of the CoCos outstanding were eligible as AT1 and in 2015 about 76% were AT1 CoCos. The greatest amount of CoCos issued worldwide is in Europe, hence this is the appropriate environment to study CoCos and WDs issues. The conclusions of this study are relevant for both the US and European regulators, financial decision-makers and investors. What is the impact of

introducing hybrid capital like CoCos and WDs into banks' financing structure on their risk appetite and medium and long-term investment decisions? The only existent comprehensive empirical study on CoCos is conducted by Avdjiev et al. (2015). They interestingly show that CoCos issuance reduces banks' credit risk and investors in CoCos view those instruments as risky and place a significant likelihood on the possibility of conversion.

Chapter 1

How to shape risk appetite in presence of franchise value?[2]

Abstract

We propose a model where risk appetite is determined by the interplay of the default put option and the down-and-out call option, pricing the franchise value. The bank manager takes incremental decisions maximizing his objective function, i.e. the sum of the two options, adjusting jointly the level of leverage, assets and franchise value volatility and the policy rate. Risk appetite is given by the first order derivatives. We show that regulators should tune their recommendations depending on the targeted cluster, since the driver of risk appetite alternates between the two options depending on the cluster and on the underlying variable considered. We find that the optimal policy rate for stability is higher with respect to the existing one in the last period. The modelling framework and the insights emerging from the cluster analysis are our major contribution to the existent literature.

Keywords: risk appetite, policy rate, default put option, down-and-out call option, franchise value, assets and franchise' volatility, leverage.
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[2]We acknowledge the financial support of the Europlace Institute of Finance (EIF) and the Labex Louis Bachelier (Grant 2014)

1 Introduction

The main goal of our research is to understand bank's risk appetite. The modelling framework and the insights emerging from the cluster analysis are our major contribution to the existent literature. We find what are the main drivers for risk appetite and their impact on both the bank market value and its franchise value. Bank risk appetite is determined by the interplay of the default put option, and the franchise value, i.e. the net present value of non-observable bank's growth opportunities, priced through a down-and-out call option. The franchise value is not directly observable: how can we evaluate this figure? We rely on Barone-Adesi et al. (2014) pricing it through the down-and-out call option and we propose to estimate it implicitly from the equity market value, extending the standard structural models. Risk appetite is determined by the manager objective function which we propose it to be the sum of the two options, since these are the key determinants of the bank market value. We propose two ways to reach the goal of shaping risk appetite. First, we suggest a two steps optimization problem, second, we assess risk appetite in a state space model. In the first step of the optimization problem, we estimate the non observable quantities, namely the franchise value and the market value of the assets. The optimization procedure is based on the non-linear least squares estimator, through which we minimize the distance between the equity market value figures and our model. We discriminate banks with and without franchise value and subsequently we perform a cluster analysis based on leverage. Given the industry in which we perform our analysis, we rely on the Basel Committee on Banking Supervision definition for leverage[3], i.e. the ratio between the tier one and the total exposure. In our model, the total exposure is given by the franchise value and, thus, the market value of the assets. The Federal Reserve announced that the minimum Basel III leverage ratio would be 6% for systemically important financial institution banks and 5% for their insured bank holding companies. Hence, we encounter those thresholds as reference point given that our empirical analysis is based on US banks. In the second step, we optimize the objective function, given by the sum of the two options, evaluating risk appetite simultaneously with respect to leverage, volatility and the policy rate. The first-order derivatives of this function determines the bank risk appetite. The determinant of the Hessian matrix tells us in which direction the manager optimizes this function. In our model, the bank manager should align her policy within the regulator framework. The bank manager sets the Vega[4] equal to zero (volatility-driven risk appetite) simultaneously with the derivative with respect to the leverage (leverage-driven

[3] We refer to Basel III definition.

[4] Objective function pricing sensitivity with respect to change in the implied volatility.

risk appetite), and with the rho[5] equal to zero (policy rate-driven risk appetite). We stress the importance of performing simultaneously this optimization among the three variables, since, in our framework, it is important their joint impact over the bank risk appetite. The second order and joint derivatives are relevant in the optimization procedure, since the sign of the determinant of the Hessian matrix gives us the direction of the bank policy. The manager has discretionary power over volatility and leverage. Nevertheless, we focus also on the optimal value of the policy rate in order to understand the level that would grant stability for each bank and, thus, for the whole banking system. In our case, it is the rate that, together with the optimal values for the other two variables, maximizes the bank value. We stress the difference between our optimal policy rate and the risk-free rate given by the monetary policy. The latter is determined by a number of other factors that are not the subject of this paper. The pricing of the two options is designed to have different impact on the appetite for risk of our bank depending on the cluster we focus on and the variable we consider. The regulators should tune their recommendations depending on the targeted cluster in order to be effective. This is an element of primary interest because regulation does not differentiate enough in the banking industry and flat recommendations do not fit all the peculiarities we find in clustering the industry. Our specification of objective function returns a three-dimensional perspective and addresses the main instruments of regulation. Thus, it can be a useful instrument for the regulator, allowing for a more comprehensive understanding of the joint impact of the three optimizing variables.

Our empirical sample consists of 1436 listed US banks and the time span considered is 1980-2014. We perform a cluster analysis in order to accommodate for the main differences across the industry. First of all, we distinguish between banks with franchise value and without. We find that about the 15% of the banks in the sample do not have franchise value at least in one year of the time span considered. Second, we cluster our sub-samples into three categories depending on leverage. The main results for the sub-sample of banks without franchise value are easy to predict since the put option is the only player in the objective function optimization and in determining the shape of risk appetite. More interestingly, we assess the sub-sample of banks with a portfolio of growth opportunities and we cluster it as follows: (i) “over-capitalized” banks (cluster 21), with an actual average leverage of 11.12%, (ii) “average capitalized” banks (cluster 22) with 7.41% and (iii) “under-capitalized” banks (cluster 23) with 4.28%.

We perform a sensitivity analysis with respect to the three optimizing variables in order

[5] Objective function sensitivity with respect to change in the policy rate.

to understand the shape of risk appetite moving one of the three variables given the optimal quantities for the other two. Furthermore, we disentangle which option is the main driver among the three clusters[6]. Considering leverage, the risk appetite is determined first by the default put option, then by the down-and-out call one. There is a difference in the leverage-driven risk appetite at a cluster level related to the positioning of the peak. On the volatility side, both the options contribute in shaping the objective function but the default put option is an early operator with respect to the down-and-out call one. As the leverage decreases, the volatility-driven risk appetite is smaller, since the risk appetite peaks goes to the left-hand side. Concerning the policy rate, the shape of risk appetite is a concave function in all the three cases, with minor differences among the clusters. The down-and-out call option drives the shape at the beginning leaving the place to the put one afterwards. In cluster 21, the bell-shape is quite symmetric, instead in the other two it is right-skewed. Empirically, we always find that the estimated policy rate is higher relative to the actual one in the last period. Increasing the leverage, the optimized objective function naturally decreases (since it is partly determined by the franchise value, but this is due to our definition of leverage). The risk appetite is assessed through the behaviour of its three main drivers and the associated shape is quite different among the clusters.

The paper proceeds as follows. Section 2 introduces a literature review. Section 3 presents the model and the pricing of the options. The two step optimization problem is described in Section 4. Section 5 shows the empirical results. Section 6 concludes. Further material is given in the appendix.

2 Literature review

This paper is related to several different strands of the literature. First of all, the building blocks of the literature about structural models are considered. Second, regulation issues are reviewed from both a theoretical and an empirical perspective. Third an overview on growth opportunities evaluation issues is presented.

Our paper is based on the seminal works by Black and Scholes (1973) and Merton (1973a), where the liabilities of a company are seen as an European option written on the assets of a firm. In the case of Merton (1973a), the capital structure of a firm is composed by a zero-coupon bond, as debt, and equity. At the beginning of the period, debt holders hold a

[6]These results are in line with the signs of the first order derivatives of the objective function with respect to the optimizing variables.

portfolio consisting of the face value of debt and a short position on a European put option. Instead, equity holders hold a European call option on the market value of assets, with strike equal to the face value of debt. Under the non arbitrage assumption, the price of this option is equal to the market value of equity. Default can happen only at maturity and standard Black-Scholes world assumptions[7]hold.

Several studies extend the original model considering the assumptions by Black and Scholes. Black and Cox (1976) and Longstaff and Schwartz (1995) allows for default also prior to maturity. Merton (1977, 1978) examines default risk in banks, with several issues that were addressed by recent literature. In those cases, equity is considered as a barrier option and the default event is triggered at the first hitting time of an exogenously determined barrier.

The endogenization of the default threshold, proposed by Leland and Toft (1996), provides alone not a clear improvement with respect to the standard Merton model, unless a jump component is introduced, as in Leland (2006). Our study is more related to Brockman and Turtle (2003), who introduce in equity path dependency, i.e. equity can be knocked out whenever a legally binding barrier is breached. We assess the market value of equity building on Babbel and Merrill (2005). In their model, the franchise value and the default put option accrue to equity holders Barone-Adesi et al. (2014) argue that the risk appetite of financial intermediaries is determined by the interplay of default put option and growth opportunities.Our contribution to the existent literature is the modelling framework we propose. Starting from those seminal studies, we assess risk appetite in a three-dimensional framework, in order to understand the joint impact our three variables have on each other and on the bank risk appetite.

The model we propose relies on regulatory principles: Basel III indicates a bank is insolvent if the common equity tier one (from now on CET1) is below 4.5%. It is true that in some countries banks, that would be declared insolvent for Basel III, still run their assets. Thus, a possible extension to this model would consider the interplay between an exogenous default barrier set by the regulator and the endogenous one chosen by the bank, highlighting an important weakness in monitoring by the regulator.

From a social point of view, Hugonnier and Morellec (2017) provide a measure for social benefits of regulation, in order to avoid the burden of a bank default to be beared by the taxpayers. It would be interesting to extend our model to the too-big-to-fail banks. In this case, Lucas and McDonald (2006) build their modelling of the public guarantee in a Sharpe

[7]Such as perfect markets, continuous trading, constant volatility, deterministic and constant interest rates, infinite liquidity and Ito dynamics for the process of the market value of the assets in place

(1976) and Merton (1977) framework, where the insurance is a put option on the assets' value. For a firm with guaranteed debt, equity value has another component with respect to the standard call option on the operating assets: the public guarantee. It is assumed to accrue to equity holders, since it is equivalent to writing a put option, from the government point of view.

Another main ingredient in our study is franchise value, which is the net present value of future growth opportunities.

The underlying framework, for our model, is given by Froot and Stein (1998) who found the rational for risk management arises from the concavity of the franchise value.

3 Research methodology

3.1 The model

The subject of our study is a bank held by shareholders who benefit from limited liability. They discount cash flows at a constant interest rate.

The structure of the balance sheet, in book values, is given as follows. The bank owns a portfolio of risky assets and liquid reserves, and is financed by insured deposits, risky debt and equity. On the left hand side of the balance sheet, risky assets are relative illiquid due to informational problems (see e.g. Hugonnier and Morellec (2017) and Froot and Stein (1998). For the while, assuming there are no costs of raising funds, liquidity reserves do not play a role. On the right hand side of the balance sheet, the focus of the analysis is on risky debt and equity, instead deposits are seen as a relative stable source of financing for the bank (see Hanson et al. (2014)).

Going to market values, debt is seen as a portfolio of cash plus a short position in a put option on firm value as in Merton (1974) and equity as a call option on assets as in Black and Scholes (1973). In our model, we focus on the interplay between the standard default put (PUT^{def}) option and the down-and-out call (DOC) option that accrue to shareholders.

Main assumptions and model description

In this subsection, we introduce the main assumptions of our model, building on the fundamental work of Black and Scholes (1973) and Merton (1974), and the following insights by Babbel and Merrill (2005) and Barone-Adesi et al. (2014).

The setting of the underlying model deals with continuous time, with initial date $t = 0$ and terminal date $t = T$. No frictions, like transaction costs, taxes and costs of raising funds, nor limits on short sales are considered and no riskless arbitrage opportunities exist. Agents are risk-neutral and there are no conflicts of interest between shareholders and managers. The focus of the project is to understand how the regulator should set appropriate risk-taking incentives, given that the bank is maximizing its end of the period equity market value.

Initially, shareholders contribute the entire equity of the bank and, subsequently, consider operating a debt-equity swap at t_0, where debt has face value FV^{SD}. The proceeds from debt issue are invested in the assets in place and future growth opportunities (i.e. franchise value) that at time T are worth $A(T)$ and $Fr(T)$, respectively. The franchise value materializes only at the end of the period, T, but it might vanish previously, as soon as the liabilities exceeds the asset value in $0 \leq t \leq T$, that is when

$$\tau_{Fr=0} = inf\left\{t \geq 0 : A(t) \leq FV^{SD} + Dep\right\}. \tag{1}$$

This is slightly different with respect toDemsetz et al. (1996) or Jones et al. (2011), because in their model this value is lost in case of bankruptcy. We call the market value of the assets (MVA) the sum of the tangible value of the assets and franchise value. Their dynamic is:

$$d\ln(MVA(t)) = \left(\mu_{MVA_t} - \frac{\sigma^2_{MVA}}{2}t\right)dt + \sigma_{MVA}dB_t, \tag{2}$$

where B_t is a standard Brownian motion, the drift, μ_t, is time-varying and σ is constant and both are referred to the sum of the tangible value of the assets and the franchise value.

The default can occur only at the end of the period, T, in case liabilities exceed assets.

For simplicity, we fix the risk-free rate and dividend issues equal to zero. Similar to Babbel and Merril (2005) and Barone-Adesi, Farkas and Medina (2014), we split the value of the bank into three components. First, considering the limited liability, the market value of the equity of our bank is a call option on the assets:

$$E(T) := max(A(T) - L), \tag{3}$$

where A is the value of the banks' assets and L the face value of the liabilities. Second, let's split the value of equity into the following two components:

$$E(T) := X(T) + Put^{def}(T), \tag{4}$$

where $X(T) := A(T) - L$ is the net tangible value of the bank, without considering the limited liability, which is represented through the default put option. Third, we allow the bank to be able to invest in value creating opportunities at time T, through the introduction of the franchise value ($Fr(T)$). Hence,

$$E(T) := X(T) + Put^{def}(T) + Fr(T). \tag{5}$$

The bank's balance sheet at time zero can be summarized as follows:

Assets	Liabilities and Net Worth
	Deposits: $D(0)$ Short term liabilities: $SL(0)$ Long term Liabilities: $LL(0)$
Tangible Assets: $A(0)$	
Default Put Option: $Put^{Def}(0)$ DOC Option: $DOC(0)$	
Intangible Assets: $Put^{Def}(0) + DOC(0)$	**Total Liabilities:** $D(0) + L(0)$
	Shareholder Equity: $MVE(0)$
Total: $A(0)+Put^{Def}(0)+DOC(0)$	**Total:** $D(0) + L(0) + MVE(0)$

Table 1: Bank balance sheet at time zero.

Taking into account the different sources of financing for our bank (deposits, standard debt and CoCos), the end of the period equity market value is given by the three components: the net tangible value, the shareholders' option to default and the franchise value.

$$MVE(T)=\begin{cases} A(T)+Fr(T)+Put^{def}(T)-L(T)-D(T) & if \quad MVA(T)\geq(L(T)+D(T)) \\ (L(T)+D(T))-A(T) & if \quad MVA(T)<(L(T)+D(T)) \end{cases} . \tag{6}$$

In the expressions above, we show that the franchise value come to fruition in case the tangible value of the assets do not fall below the contemporaneous value of the liabilities and deposits. We comment on the barrier in the context of the DOC pricing. Furthermore, in case the franchise value do not vanish, the put option is out of the money and the shareholders do not exercise the put option and its present value is still given by the option price that can be potentially exercised in the future. The opposite is true when the tangible value of the assets is eroded. In order to clarify the economic interpretation of the table above, we show here the terminal claim on the DOC option is:

$$DOC(T)=\begin{cases} F(T) & if \quad MVA(T)\geq(L(T)+D(T)) \\ 0 & otherwise \end{cases} ; \tag{7}$$

At time zero, equity market value exceeds the capital supplied by the shareholders and this

difference comes from the value at time zero of the franchise value and the option shareholders have to default. We give the pricing of those options in the following sections.

3.2 Pricing the default option

Following the reasoning in Barone-Adesi et al. (2014), bank shareholders are long on the default option, which the manager has to maximize acting on behalf of the shareholders. The pricing formula for the value of the put option at time zero together with the DOC one we present in the following section, considers the franchise value as major ingredient both in the underlying value and in the volatility. The franchise value has to be taken into account in the market value of the assets. This is necessary in order to prevent potential arbitrage opportunities, that could arise otherwise, buying the bank and selling short the tangible assets and the franchise value, if this last one would not be considered. The put option is a convex, decreasing function of the asset value and is maximized when the value of the liabilities, as well as the riskiness of the assets is magnified. This means it is a a driver for risk-taking. The underlying is given by MVA, the value of the net tangible assets at the beginning of the period, and Fr the franchise value. The strike price is the market value for straight debt, MV^{SD}[8]. T is the time to maturity, σ_{MVA} is the volatility of both the assets and the franchise value and rf is the policy rate. From this paragraph on, we relax the hypothesis regarding the risk-free rate, allowing it to fluctuate both in the positive and negative domain. We define leverage following Basel regulators criterion, i.e. the ratio between the $Tier1$ and the total exposure, which we subsume in the market value of the assets considering this way both the value of the tangible assets and the value of future growth opportunities. Our pricing of the default put option is given by:

$$
\begin{aligned}
Put^{def}\left(lev, \sigma_{MVA}, rf\right) \quad &= \left(MV^{SD} + D\right)\Phi\left(-d_2\right) + \\
&\quad \left(-\left(MVA\right)\Phi\left(-d_1\right)\right), \\
&\quad \text{with } \{\tau_{Fr=0} > T\}, \\
where \qquad & d_1 = \left(\frac{ln\left(\frac{1}{1-lev}\right) + \left(rf + \frac{\sigma^2_{MVA}}{2}\right)T}{\sigma_{MVA}\sqrt{T}}\right), \\
lev = \left(\frac{Tier1}{MVA}\right), \qquad & d_2 = d_1 - \sigma_{MVA}\sqrt{T},
\end{aligned}
\tag{8}
$$

We consider without loss of generality Φ the standard Normal. In absence of growth opportunities, the pricing formula goes back to the standard one. The greeks for this option are given as follows:

$$
\begin{aligned}
Sensitivity \;\; to \;\; leverage \;\; &: \left[\frac{\delta Put^{def}_{i,t}}{\delta lev_{i,t}}\right] < 0 \\
Sensitivity \;\; to \;\; volatility \;\; &: \left[\frac{\delta Put^{def}_{i,t}}{\delta \sigma_{MVA_{i,t}}}\right] > 0 \\
Sensitivity \;\; to \;\; policy-rate \;\; &: \left[\frac{\delta Put^{def}_{i,t}}{\delta rf_{i,t}}\right] < 0.
\end{aligned}
\tag{9}
$$

This option push the bank manager to adopt a risk-taking policy. We present further infor-

[8]We proxy the market value for the straight debt with the KMV model (KMV corporation).

mation about the sensitivity of the default put option with respect to volatility, leverage and policy rate in the optimization section.

3.3 Pricing the DOC option, in presence of non-observable underlying

At time T, $F(T)$ represents a portfolio of positive net present value growth opportunities. Before maturity, the expected value of $Fr(T)$ is embedded in the value of the risky assets of the bank and is the franchise value which is given by the DOC option (see Barone-Adesi et al. (2014)). This option is a down and out call, with a pricing formula that is slightly different from a mathematical point of view with respect to the standard one in Black-Scholes framework (Merton (1973a)), but it confers a much different economic interpretation, where $Fr(0)$ constitutes the value of potential growth net of investment cost in the case the bank does not opt for default. Since investment costs are already considered in $Fr(0)$, the strike for this option is set to zero. The barrier is given by the market value of standard debt and deposits. This option is priced in an European framework given that the franchise value comes to fruition only at maturity[9], but it is path dependent. In case the barrier is breached before maturity the option expires and the franchise value is driven immediately to zero. The extended standard pricing is given as follows:

$$\begin{aligned} DOC\left(lev, \sigma_{MVA}, rf\right) \quad & = Fr\left[\Phi\left(v_1\right) + \right. \\ & \left. - \left(1 - lev\right)^{2\lambda} \Phi\left(y_1\right)\right] \\ & with \left\{\tau_{Fr=0} > T\right\}, \\ where \quad & \lambda = \frac{rf + \frac{\sigma^2_{MVA}}{2}}{\sigma^2_{MVA}} \end{aligned} \tag{10}$$

$$v_1 = \frac{ln\left(\frac{1}{1-lev}\right)}{\sigma_{MVA}\sqrt{T}} + \lambda\sigma_{MVA}\sqrt{T}, \quad y_1 = \frac{ln(1-lev)}{\sigma_{MVA}\sqrt{T}} + \lambda\sigma_{MVA}\sqrt{T}$$

The franchise value, $Fr(0)$, is not directly observable. However, we present below how to estimate it in the framework of our model. Indeed, the value of the DOC option is a part of the market value of the assets, where the remaining is given by the tangible assets. The term in parenthesis gives the pricing probability that the intermediary will survive long enough for

[9] that is equivalent to say that we can exercise it only at maturity

the growth opportunities to come to fruition. The standard greeks for this option are given as follows:

$$\begin{aligned} Sensitivity \quad to \quad leverage &: \left[\frac{\delta DOC_{i,t}}{\delta lev_{i,t}}\right] > 0 \\ Sensitivity \quad to \quad volatility &: \left[\frac{\delta DOC_{i,t}}{\delta \sigma_{MVA_{i,t}}}\right] > 0 \\ Sensitivity \quad to \quad policy-rate &: \left[\frac{\delta DOC_{i,t}}{\delta rf_{i,t}}\right] > 0 \end{aligned} \tag{11}$$

We show in the following sections when the DOC prevails over the default put one in determining the shape of objective function and consequently the one of risk appetite.

3.4 The optimization problem for risk appetite

Risk appetite is a non-negative real number that describes investor's appetite for risk, with higher values corresponding to a greater degree of aggression. Risk appetite is commonly defined as the level and type of risk a firm is able and willing to assume in its exposures and business activities, given its business objectives and obligations to stakeholders. We prefer to understand the appetite for risk of the bank, rather than concentrate on risk-taking, because, in the definition we propose, the default put option promote risk-seeking instead the DOC one is designed to refrain the bank to undertake excessive risk. A rising risk appetite implies that investors are willing to hold riskier assets, obtained through assets' substitution. Since it is not possible to observe directly risk appetite, we need to understand how it is determined and where to extract information about its manifestation. From both a risk-management and a regulation point of view, it is a priority to infer some information about objective function in the banking system.

In case the market value of the assets exceeds the bank liabilities, the two determinants of the market value of the equity are the down-and-out call option and the default put option. Hence, we define the manager objective function ($O.f.$) as the sum of the two options:

$$O.f._{i,t} := DOC\left(lev_{i,t}, \sigma_{MVA_{i,t}}, rf_{i,t}\right) + PUT^{def}\left(lev_{i,t}, \sigma_{MVA_{i,t}}, rf_{i,t}\right). \tag{12}$$

We investigate the risk appetite of the bank over three variables: leverage, assets and franchise value volatility and the policy rate. The bank manager has decision power only over the first two, but we aim to understand the optimal policy rate which should grant stability. The risk appetite is determined in the optimization problem we present in Section

5.2 and is given by the first order derivatives and the determinant of the Hessian matrix.

The optimization problem is twofold because in the first step we estimate the franchise value and the market value of the tangible assets that are not observable in the market, but are embedded in the equity market value. Those two variables are key in order to perform the second optimization step, where we look for the optimal level of leverage, assets and franchise value volatility and policy rate, that simultaneously optimize the objective function.

3.4.1 First step ingredients

In the first step, the goal is to estimate the unobservable franchise value and the consequent market value of the assets. We argue that our unobservable quantities are embedded in the equity market value. We model the bank equity market value through the put-call parity, as the sum of the call option on the assets, the default put option and the franchise value, priced as the DOC option, considering the value of the bank at the time in which the franchise value comes to fruition.

At the beginning of the period, we consider the following system of equations:

$$\begin{cases} MVE_{i,t} = & (A_{i,t} - (MV^{SD}+D)_{i,t} + DOC_{i,t} + Put^{def}_{i,t}), \\ \sigma_{MVE_{i,t}} MVE_{i,t} = & \sigma_{MVA_{i,t}} (MVA_{i,t}) \Phi(d_{1i,t}), \\ where \quad d_{1i,t} = & \left(\dfrac{ ln\left(\frac{MVA_{i,t}}{(MV^{SD}+D)_{i,t}}\right) + \left(rf_{i,t} + \frac{\sigma^2_{MVA_{i,t}}}{2}\right) T }{\sigma_{MVA_{i,t}} \sqrt{T}} \right) \end{cases} \tag{13}$$

This extension of the Merton specification allows us to consider the franchise value both at the underlying and implied volatility level. Since the equity market value incorporates the information regarding both the assets market value and the franchise value, consequently the implied volatility estimated in this model refers to the one considering both the assets and the franchise value. At the beginning of the period we do not have information regarding the franchise value, so that we consider its price through the DOC option. We solve this problem through the non-linear least squares criterion function, for each bank at any time t on the whole time span considered, optimizing the distance between the data concerning the equity market value and the model extended accommodating for both the default put option and the DOC one. We perform a step by step optimization for $\Theta_{i,t} := Fr_{i,t}, A_{i,t}, \sigma_{MVA_{i,t}}$, building on the Bellman's Principle of Optimality (Bellman (1952)), applied also in Merton (1973b).

In order to perform this, we build on the following error function, solving simultaneously:

$$\begin{cases} e_{1,i,t}= & MVE_{i,t}-(A_{i,t}-(MV^{SD}+D)_{i,t}+DOC(\Theta_{i,t})+Put^{def}(\Theta_{i,t})), \\ e_{2,i,t}= & \sigma_{MVE_{i,t}}MVE_{i,t}-\sigma_{MVA_{i,t}}MVA_{i,t}\Phi(d_{1i,t}), \end{cases} \tag{14}$$

where $\{i\}_1^n$ is the bank identificator and $\{t\}_1^m$ the year considered. In this specification, we perform our analysis at the beginning of the period, because we need to estimate the major ingredients for the objective function optimization. The non linear least square function is the following:

$$\Theta^*_{i,t} = \underset{(\Theta_{i,t})}{arg\min} \quad \sum_{j,i,t=1}^{2,n,m} \left[e^2_{j,i,t}\right] \tag{15}$$

where the optimal quantities are $\Theta^*_{i,t}$, that optimize the sum of the squared deviations, which are the non-linear least squares estimators. Thus, we can proceed to the next step optimizing the objective function[10]. At this step, empirically, we proceed in our first clustering distinguishing among banks with franchise value and without.

3.4.2 Second step

In this step, we optimize the objective function to cope with the regulator standard indications concerning leverage and assets' volatility. Furthermore, we derive also the optimal policy rate for the stability, first, of each bank, and second, of each cluster, aggregating together the results. The manager optimizes the objective function of the bank, modifying its exposure to risky assets and adjusting bank's leverage at time zero, operating always for allowing the franchise value to come to fruition at time T. The shape of risk appetite is assessed through the determinant of the Hessian matrix in a three-dimensional perspective. We propose a volatility-driven risk appetite, as well as a leverage-driven one and a policy rate-driven one. The outline of those optimal quantities differs among clusters depending on which option drives the behaviour in that specific case. This is an element of primary interest because regulation does not differentiate enough in the banking industry and flat recommendations do not fit all the peculiarities we find in clustering the industry. Furthermore, the impact can be counter-productive, given that differences among clusters are relevant and consequences can go in an opposite direction with respect to what is intended by the regulator.

[10] As we explain in the following step, we perform the optimization at each time step t, following Bellman (1952) and Merton (1973b), in order to allow the franchise value of the bank to come to fruition at time T.

In our framework, where the objective function is driven by the two options, we insist in setting also *rho* equal to zero, focusing on the sensitivity of the objective function with respect to the policy rate (policy rate-driven risk appetite). This is relevant to understand the joint impact of the three variables and, thus, to derive an optimal rate which should grant stability at both single bank level and at cluster level, and consequently, for the whole banking system. The decision variables over which, instead, the manager has discretionary power at time zero are volatility and leverage. On the bank manager side, the shape of risk appetite is determined setting equal to zero *vega* (volatility-driven risk appetite) and the first order derivative with respect to the leverage (leverage-driven risk appetite) [11]. All of those first order derivatives are obtained given optimal values for the other two variables. Our optimization procedure goes beyond what presented till now. We accommodate for a joint optimization, where the three optimal quantities are estimated simultaneously The optimization variables are the leverage, the assets' volatility and the policy rate, so our theta in this case is: $\Theta_{i,t} := \left(lev_{i,t}, \sigma_{A_{i,t}+Fr_{i,t}}, rf_{i,t}\right)$. When $MV^{SD} < (MVA\,(0) + Fr\,(0))$, the optimization problem is:

$$\Theta^*_{i,t} = \underset{\Theta_{i,t}}{arg\max} \quad [Of_{i,t}] \tag{16}$$

In this framework our three-dimensional risk appetite (R.A.) is given by:

$$\begin{aligned}
&leverage - driven \quad R.A.: \quad \left[\frac{\delta Of_{i,t}}{\delta lev_{i,t}}\right] = 0 \mid_{\sigma^*_{A_{i,t}+Fr_{i,t}}, rf^*_{i,t}} \\
&volatility - driven \quad R.A.: \quad \left[\frac{\delta Of_{i,t}}{\delta \sigma_{A_{i,t}+Fr_{i,t}}}\right] = 0 \mid_{lev^*_{i,t}, rf^*_{i,t}} \\
&policy - rate - driven \quad R.A.: \quad \left[\frac{\delta Of_{i,t}}{\delta rf_{i,t}}\right] = 0 \mid_{lev^*_{i,t}, \sigma^*_{A_{i,t}+Fr_{i,t}}}
\end{aligned} \tag{17}$$

Numerically, we use the methodology developed by Byrd et al. (1995) which allows box constraints, that is each variable can be given a lower and/or upper bound. The initial value must satisfy the constraints. This uses a limited-memory modification of the BFGS quasi-Newton method (Broyden (1970); Fletcher (1970); Goldfarb (1970); Shanno (1970)). The algorithm always achieve the finite convergence.

In presence of interest rate risk, diversification provides an additional risk management opportunity. Indeed, if the interest rate and asset risk exposures are of similar magnitude,

[11] In standard literature, it does not exist a "greek letter" identifying the sensitivity of an option price with respect to leverage.

and if these risks are uncorrelated, then one would expect diversification to be very important, especially if franchise values are high. In this case we perform a pointwise optimization since we are interested in the parameters that optimize the objective function of each bank on the whole time span. The optimal objective function do not have theoretical bounds, but we focus on $0 \leq O.f._{i,t} \leq 1$, since it is hard to find empirically a bank having the sum of the two options greater than the market value of the assets (our normalizing quantity). Although it is well known what is the behaviour of the default put option with respect to the three variables assessed, the same is not straightforward for the DOC pricing and, consequently, for our specification of objective function. In the appendix we show the derivation of both the first order derivatives and the cross ones, taken into account given the simultaneous approach. In order to understand which option is the main driver for the objective function we need to perform a cluster analysis. Section 6 presents the main theoretical results before showing the empirical ones, thus it will become clearer the shape of the objective function and the consequent risk appetite one. Our differentiation among clusters is crucial for setting effective regulatory recommendations, because flat rules miss the peculiarities of the different patterns of objective function we could appreciate in the clustering. In section 5 we present results both aggregated and clustered, pointing out the importance of more accurate analysis in this domain.

4 Results

4.1 What drives our three-dimensional risk appetite? A simulation exercise

In our definition of the objective function, two options play a role. When the bank does not have any consistent portfolio of growth opportunities, the DOC option is worthless so that the default put option is the only determinant of the objective function and of the risk-appetite. In this case, it is well known what is the impact of the optimizing variables and consequently what is the shape for risk appetite. But what happens when the bank has an embedded franchise value? In this case in a theoretical framework it is not clear which option has the main impact on the objective function, theoretically which is the main driver and also the optimization procedure is not trivial. We assess this issue for the banks in the sample having growth opportunities at stake clustered by leverage[12]. We cluster our sub-sample of banks

[12] We present our clustering empirical analysis in Section 6.3.

with consistent franchise value into three categories: "over capitalized" banks (cluster 21), with an actual average leverage[13] of 11.12%, "average capitalized" banks (cluster 22), with 7.41% and "under capitalized" banks (cluster 23) with 4.28%. Those figures are in line with standard literature, given our definition of leverage that is the ratio between tier 1 and the bank total exposure (i.e. the sum of the assets' market value and franchise value).

We simulate the option prices, and consequently the objective function value, building on winsorized average data per cluster. We perform a sensitivity analysis with respect to the three optimization variables in order to understand the shape of the risk appetite. We assess the shape of risk appetite moving one variable, given the optimal quantities for the other two. We can see how the optimal path changes among different clusters when considering leverage and volatility. Interestingly, when looking at the policy rate, the objective function shape is similar in the three clusters. The patterns are entirely presented in the Appendix. We perform the simulation exercise also in the negative domain for interest rates. We show those results in the Appendix as well. The negative interest rates impact negatively the optimized objective function but the *magnitudo* of this impairment in value is not that relevant relative to a similar change e.g. in volatility. It would be interesting to see the effect negative rates have on the other variables and thus assess the joint impact on the objective function.

Leverage

The put option value decreases fast when the tier 1 value increases. The DOC option price is easy to see that is also a decreasing function of leverage, defined as in Basel III, given that we put the franchise value at the denominator. In the first cluster (21, i.e. "over capitalized" banks), the optimal leverage we find in the simulation is on average similar to the one we find in the empirical analysis and it decreases with the clusters. Our simulation records as optimal result the lowest level of leverage (between 5% and 6%), larger than what we find empirically and slightly above the threshold considered by the regulator, even for the cluster whose average level of leverage is below the threshold recommended.

Volatility

When considering volatility, the put options price is an increasing function, instead the DOC one is flat and relative high for smaller volatility values and decreasing afterwards. This last results looks puzzling at a first glance, since the DOC is an option and we are used to be sure about their increase in value when considering volatility. The DOC is a barrier option

[13] Actual leverage is computed adjusting to our model Basel III definition.

and when volatility is too high, there might be a breaching of the barrier and in this case we could have a sharp decrease to zero of the option value. The optimal objective function shape is determined for smaller values of sigma by the default put option and by the DOC one for larger values of our variable. As the leverage decreases, going from cluster 21 to 23, the peak for the volatility-driven risk appetite is smaller, since the objective function peaks goes to the left-hand side. This is the case because a smaller tier 1 means that our barrier is much higher and I need smaller values of volatility in order to be sure not to cross the barrier.

Policy rate

When assessing the policy rate change impact on the two options prices we show that the default put one is a decreasing function, instead the DOC option is an increasing function. This is due to the design of the option pricing. Risk appetite shape is a concave function in all the three cases, with minor differences. In the case of cluster 21, the two options have almost the same impact in determining objective function, the curve is almost symmetric. In the cluster 22 and 23, the main driver is the default put option since the risk appetite shape is skewed to the right. Overall, the optimal policy rate level results slightly high because in our model we do not consider for the while economic growth in a comprehensive framework. This will be considered in further work because the franchise value is influenced by definition from economic growth.

4.2 Empirical results at aggregate level

Our empirical sample consists of 1436 listed US banks and the sample period is 1980-2014. Balance sheet items are taken from COMPUSTAT and considered on an annual basis. Market prices from the Center for Research in Security Prices (CRSP). Price data are taken on a monthly basis to accommodate the constant volatility hypothesis.

Summary statistics for both the input and the results at aggregate level are presented in the appendix. We perform our optimizations with several initial values in order to check we have results numerically stable. Furthermore, we calculated the confidence intervals. We show those results in the appendix as well. We find that the estimate of the risk-free rate is slightly higher with respect to the actual one especially in the last period, this is due to the fact that we do not consider economic growth. The following figure shows the evolution of both the actual policy rate (on the right y axis) and the optimal one we estimated (on the left y axis). In our model we allow for negative interest rates (in order to be coherent with the current economic situation) but the optimized values are always greater than zero.

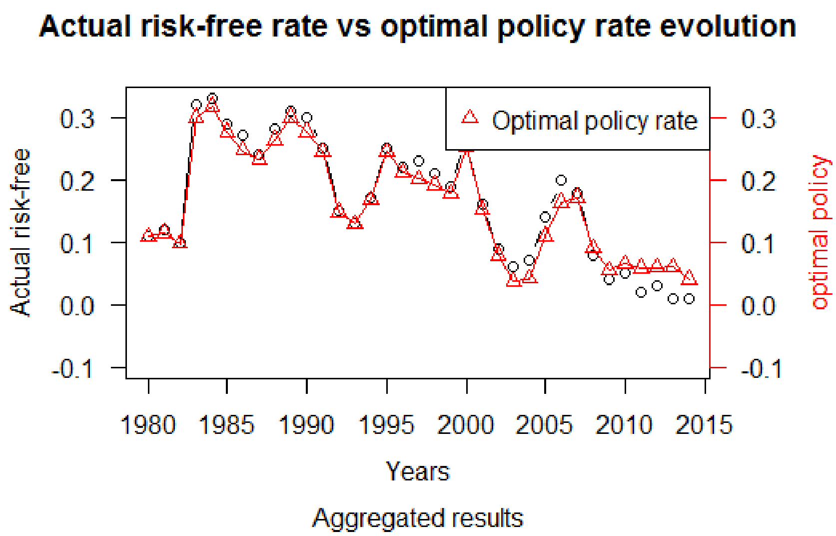


Figure 1: Optimal versus Actual policy rate evolution.

We show in the next figure that our average evaluation of the objective function aggregated on the whole sample per year and the optimal average quantities for the policy rate-, volatility- and leverage-driven risk appetite. Aggregated results loose a lot in terms of interpretability and meaning. In this aggregate dimension the objective function seems to follow the leverage pattern. On the volatility side, the two move in opposite directions. This means that some franchise value is eroded but not fully compensated by the default put option. Considering periods of lower policy rates as signaling a crisis, we can see that our measure for risk appetite is driven relative higher (that is the case after 2010). We investigate deeper the dynamic of our optimized objective function with respect to the three variables in the analysis cluster by cluster. We always represent graphically the objective function as a ratio between the sum of the two options (i.e. the objective function) and the market value of the assets, for normalisation reasons. In our model the manager chooses the optimal level of leverage, asset's volatility at the beginning of the period on the basis of past information so the present action has an impact on the following period. Our manager's policy considers the franchise value in its potential status at time t, but is backward looking, in the sense that builds on past information. The pattern is not straightforward to be interpreted, but in the time span considered, especially recently, during periods of lower interest rates optimal assets' volatility is relative higher because our manager has to shift the bank investments to riskier assets in order to perform earnings. Vice versa, during periods of relative higher interest rates we can see a flight to quality, because the bank investing in the risk-free asset is already achieving a satisfactory performance. This last consideration becomes clearer and more evident when considering clusters of banks with growth opportunities.

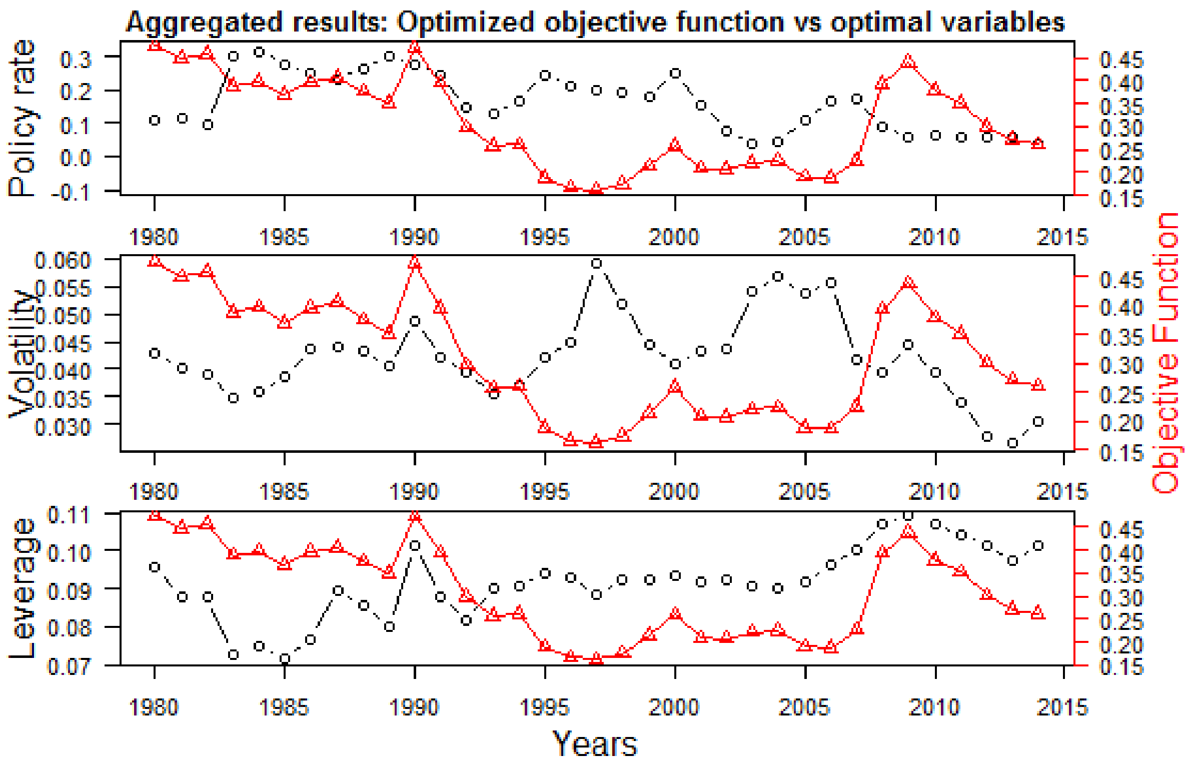


Figure 2: Objective function versus optimal variables evolution.

4.3 Empirical results in a cluster analysis

Results differ a lot when considering our cluster analysis. We perform a two step-clustering, since we first distinguish between banks with franchise value and without, second we cluster the two subsets of banks with respect to the leverage. The sub-sample of banks without franchise value accounts for about the 15% of the whole sample. In this case the optimization of the objective function and risk appetite are driven by the default put option. Thus we focus on the sub-sample of banks having the franchise value at stake. We categorize this sub-sample as follows: (i) "over capitalized" banks (cluster 21), with an actual average leverage of

11.12%, (ii) "average capitalized" banks (cluster 22), with 7.41% and (iii) "under capitalized" banks (cluster 23) with 4.28%. The input variables for our optimization and the results are presented cluster by cluster in the appendix, here we present the main results and their implications. The population of banks is not uniformly distributed across the clusters, this has an impact on quality of estimates of the sub-sample, but are a close representation of the reality.

In the following set of pictures, we can see that our estimates for the policy rate is always tracking the actual one. Only during the last years, where the actual one is driven too low by the central bank (and even inflation is zero), our estimate is relative higher because. The greater spread across the two is present in the sub-sample of banks which have a portfolio of growth opportunities (cluster 22), because our optimized objective function and consequently our risk appetite is driven upwards by the DOC option. This together with a relative low volatility lead to a very low objective function, since the bank manager optimizes his strategy investing in the risk-free asset. Banks in cluster 23, with the greatest level of debt, ask for a remarkably lower optimal policy rate especially in the last five years where the actual risk-free rate set by the regulator was at its minima levels, this is possible because of a relative lower assets' volatility necessary for franchise value preservation.

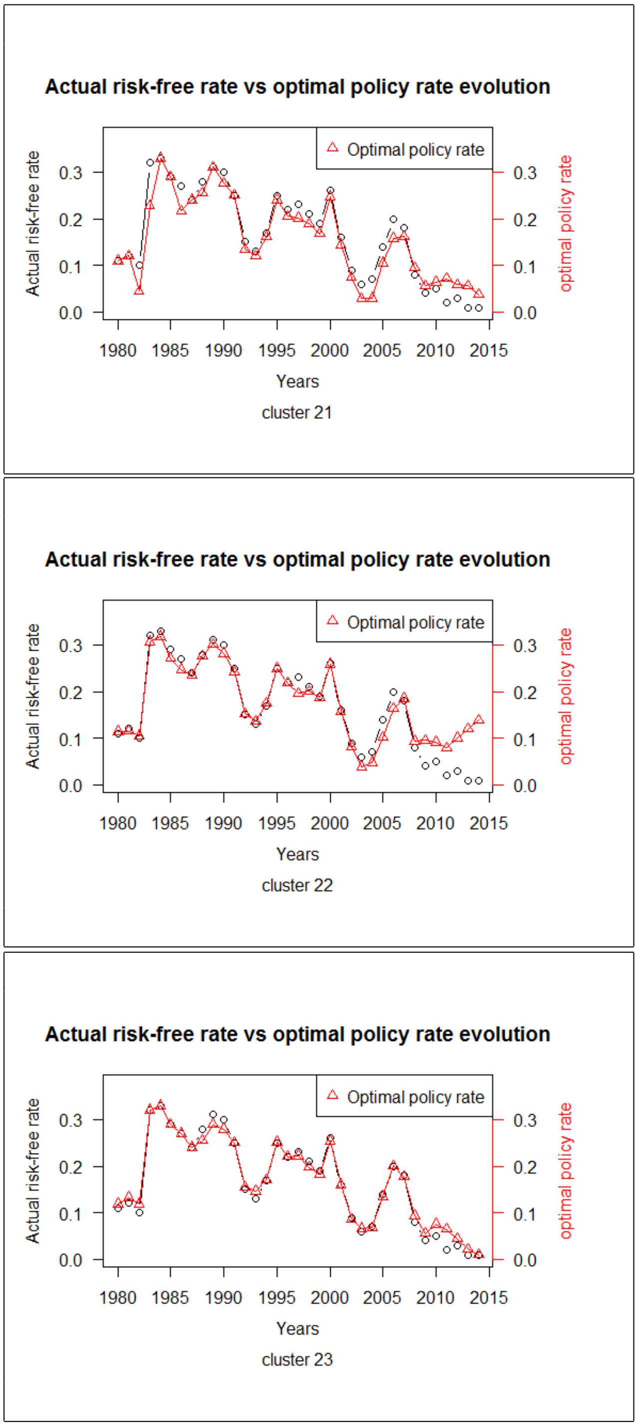


Table 2: Actual risk-free rate vs Optimal policy rate, cluster by cluster.

In the next set of pictures we present the resulting optimal estimates for the optimized objective function and its corresponding variables-driven risk appetite. Even if we do not present the results of the sub-sample of banks without franchise value, clusters 21, 22 and 23 point out results much more in line with the aggregated ones, being the greater sub-sample in terms of number of banks involved. The objective function optimal average values increases with the clusters, since we find that in cluster 21 the average level of objective function is about 16% instead in cluster 23 it approaches 41%: those clusters present smooth paths. This is not the case of cluster 22 where we can find many swings eve if the average value of the optimized objective function is about 23%. It is relevant for the regulators to take into account this evolution since banks in cluster 22 are just above the threshold recommended by the FED.

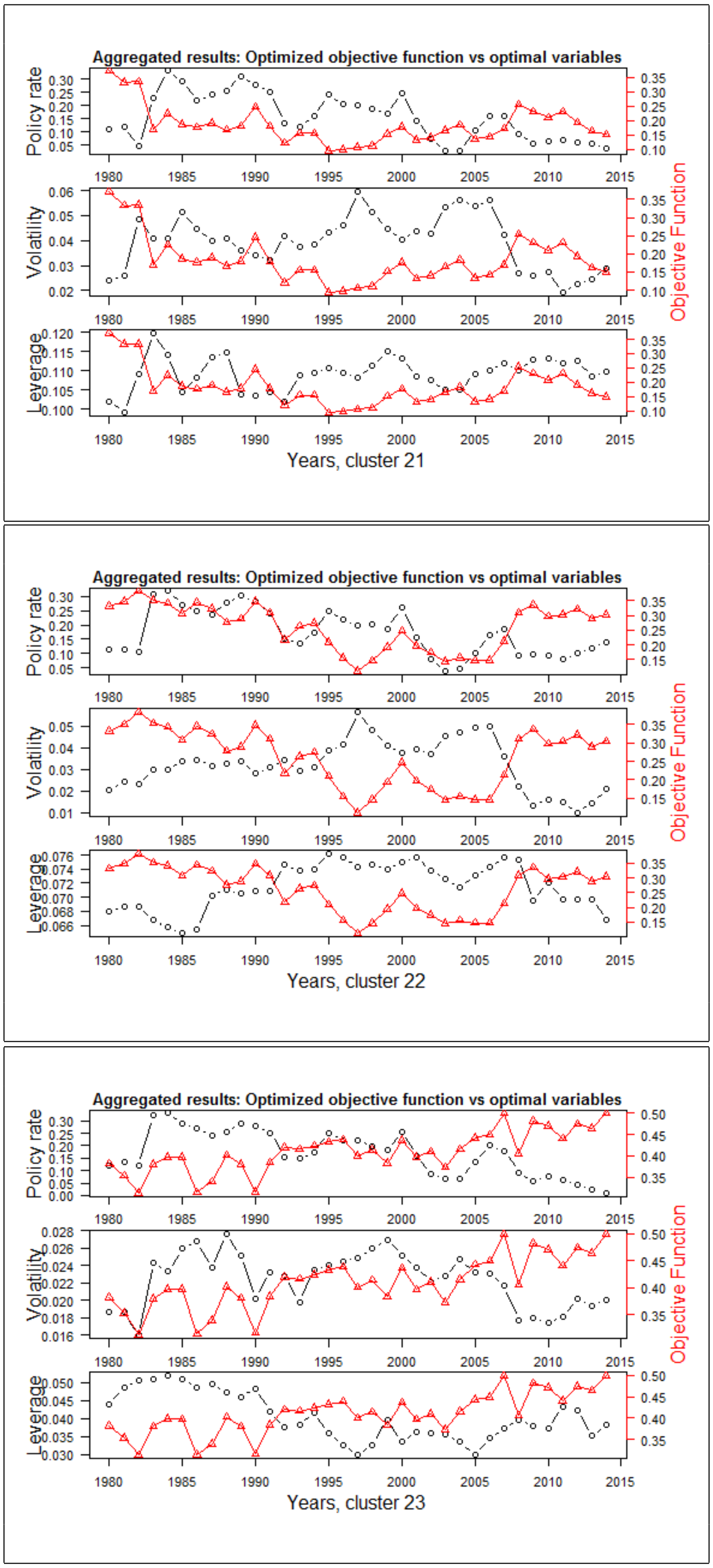


Table 3: optimized objective function evolution against its optimal determinants , cluster by cluster.

The simulation results are coherent with what we find to be optimal in this empirical analysis. On average optimal volatility is low due to the fact that on average all the clusters are characterized by a relative high barrier. At a first glance, it seems that under capitalization does not harm profitable growth opportunities, hence, the optimized objective function moves in an opposite direction with respect to leverage. Of course this is partially explained with the definition we gave to leverage, where we can find the market value of the assets and the franchise value at the denominator. On the optimal policy rate side, we find relative higher data because we should complete our model taking into account the economic growth. The evolution of the optimal policy rate moves in line with the optimized objective function, even if the latter presents wider and delayed fluctuations with respect to this variable.

5 Conclusion

In this paper, we investigate the shape of the risk appetite of our bank. Bank objective function and its risk appetite are determined by the interplay of the default option and the down-and-out call (DOC) option, pricing the franchise value, i.e. the net present value of non-observable bank's growth opportunities. We define the objective function as the ratio between the sum of the two options' prices and the market value of the tangible assets. Our major contribution consists in assessing risk appetite in a three-dimensional space, providing an insight of the importance of taking into account the joint impact of the three variables at both single bank level and cluster one.

First, we estimate the franchise value, and we discriminate banks with and without franchise value. Second, at the beginning of each period, we optimize the objective function adjusting simultaneously the level of leverage, volatility and the policy rate. The decision maker sets rho equal to zero considering also the other two variables of her risk management policy (policy rate-driven risk appetite). The bank manager sets the Vega equal to zero (volatility driven-risk appetite) simultaneously with the derivative with respect to the leverage (leverage-driven risk appetite), and the one with respect to the policy rate. Those three optimizations are conditional to the other two optimal quantities. We aim to stress the importance of the joint optimization. It results to convey a more comprehensive understanding of the bank behaviour, rather than sticking on the single elements, whose explanatory power is much more reduced.

We test our optimizations on a sample of 1436 banks, listed in the US, over 1980-2014. We find that the optimal risk-free rate is higher with respect to the existing one in the last period.

A clustering analysis is necessary in order to understand the shape of risk appetite, what are its underlying main drivers and what is the impact of changes in the optimizing variables. We show that the impact of the single variable on risk appetite is not always the same among the clusters, this is a result of both structural differences among the clusters and the joint impact of the other variables that are simultaneously optimized. The objective function is magnified for lower values of leverage, which is straightforward given our specifications and optimal volatility should stay low in order not to erode the franchise value. We find different patterns among the clusters and this imposes a cluster analysis in order to understand risk appetite behaviour. We show that regulators should tune their recommendations depending on the targeted cluster, since the driver of risk appetite alternates between the two options depending on the cluster and on the underlying variable considered, given the other two. Our three dimensional risk appetite specification could be an effective instrument for the regulator because it comprehends the three most important dimensions for shaping risk appetite in presence of franchise value. It is determined by the joint optimization, thus we need to condition on two optimal quantities in order to optimize with respect to the third one. We consider also a world where interest rates are negative, which is the case for Europe nowadays. Given our specification of objective function, negative interest rates impair the moneyness of the options but the curve are quite flat, thus *ceteris paribus* we can argue that the negative rates in our optimization procedure impact negatively on the objective function, with a limited *magnitudo*. Furthermore, introducing the franchise value in the specification of risk appetite, we propose an incentive for the manager to adopt a policy long-term oriented. There is still ample room for the regulator to find the proper instruments in order to boost banks growth on one side, and consequently help economic growth, and to prevent them undertaking excessive risks on the other side.

Appendix

A: Who drives the risk appetite? A simulation exercise

In the following table we perform a sensitivity analysis to change in the three optimization variables. We comment in Section 6 how the shape of risk appetite differs among the clusters. Optimal value ranges:

- for leverage:

$$\begin{aligned} Cluster \quad 21: \quad & 0.10 \leq lev^* \leq 0.11 \\ Cluster \quad 22: \quad & 0.07 \leq lev^* \leq 0.08 \\ Cluster \quad 23: \quad & 0.06 \leq lev^* \leq 0.07 \end{aligned} \tag{18}$$

  FED benchmarks are:0.08 for 6 Systemically important financial institution banks; 0.05 for their insured bank holding firms.

- for volatility:

$$\begin{aligned} Cluster \quad 21: \quad & 0.04 \leq \sigma^* \leq 0.06 \\ Cluster \quad 22: \quad & 0.02 \leq \sigma^* \leq 0.04 \\ Cluster \quad 23: \quad & 0.01 \leq \sigma^* \leq 0.02 \end{aligned} \tag{19}$$

  Under-capitalized banks should have a relative lower volatility: higher probability to cross the barrier and the DOC to expire.

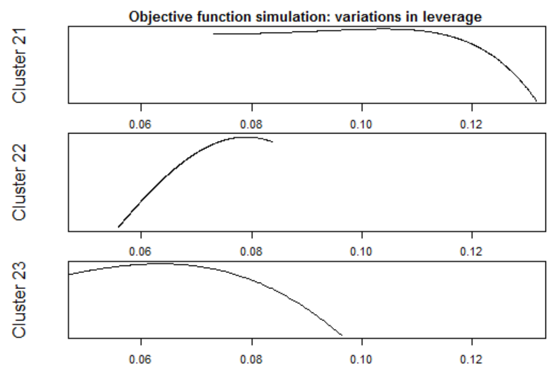
- for policy rate:

$$\begin{aligned} Cluster \quad 21: \quad & 0.13 \leq rf^* \leq 0.15 \\ Cluster \quad 22: \quad & 0.11 \leq rf^* \leq 0.14 \\ Cluster \quad 23: \quad & 0.04 \leq rf^* \leq 0.07 \end{aligned} \tag{20}$$

  We find relative higher optimal values because we do not consider economic growth and the simulation is based on input values derived from our empirical sample, time span (1980-2014).

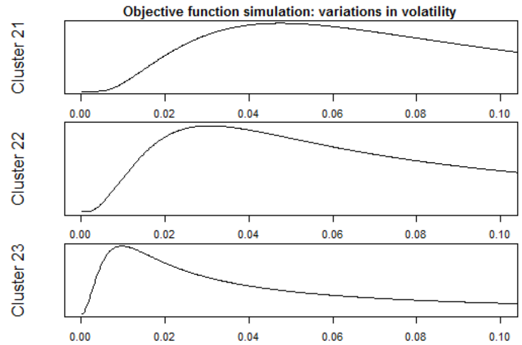
A simulation exercise: which is the main driver of the objective function between the two options? 1/3

- The optimal value for leverage in all the three clusters is above the actual levels and slightly above FED recommendation.
- Our results take into account the franchise value (in the denominator of "leverage") and this is a main difference between our results and the regulator ones.
- Given the pricing of the DOC option and the definition of leverage, the DOC is maximized for lower levels of leverage.



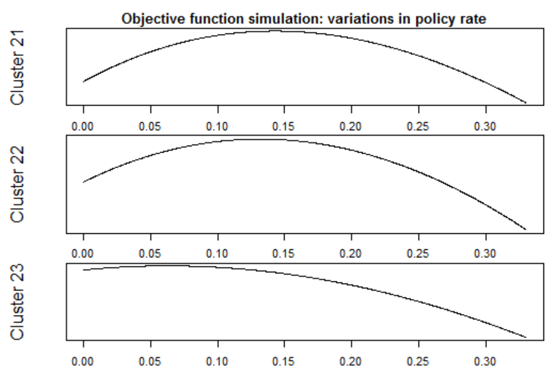
A simulation exercise: which is the main driver of the objective function between the two options? 2/3

- The DOC option, being a barrier option, is optimized in our context for relative lower volatility values: when volatility is too high, there might be a breaching of the barrier and, consequently, a collapse of the franchise value.
- With smaller values of tier1, going from cluster 21 to 23, the peak of the optimized objective functions moves to the left.
- For relative smaller tier1, our barrier is much higher and smaller values of volatility ensure the franchise value preservation.



Which is the main driver of the objective function between the two options? 3/3

- The optimized objective function displays a concave shape, with minor differences in the peak depending on the cluster.
- Once it is clear our risk appetite specification, the policy maker change in the rate has a clear impact.
- Overall, our optimal policy rate results slightly high because in our model we do not consider economic growth and inflation.



Life below zero: optimization results taking into account negative interest rates

Interest rates' decline dates back to the 1990s[14]. Since the global financial crisis, inflation has been low worldwide, and output below potential. This is the reason why in our model we do not differentiate between real and nominal rate[15]. Central bankers set policy rates at record low levels in advanced economies, and in the past few years[16], the European Central

[14] "Assessing the implications of negative interest rates", Speech by Benoit Coeure', YALE, July 28, 2016
[15] Negative real rates are have been a reality on German deposits.
[16] June 2014

Bank (ECB) became the first central bank to lower the rate of interest on their deposit facility into the negative domain. Furthermore, we take into account that also the return on government bonds is negative for most European countries[17] and Japan, even at long maturities. Hence, we need to accommodate for a negative discount rate in our model. In the previous sections, we optimize our model allowing the policy rate to range from -5% to $+33\%$ but the optimization figures are always in the positive domain and greater than zero. In this section we re-perform both the simulation exercise and the optimization for negative policy rates ranging only in the negative domain. Given that our objective function is fully characterized by the default put and DOC option, negative interest rate (in simplified terms) implies that it's less likely for the option to be in the money at expiration (for both the options) and add a discount to the option instead of a premium (on the DOC side). The major results concerning this procedure are converging for the three clusters, as we show in the following graph.

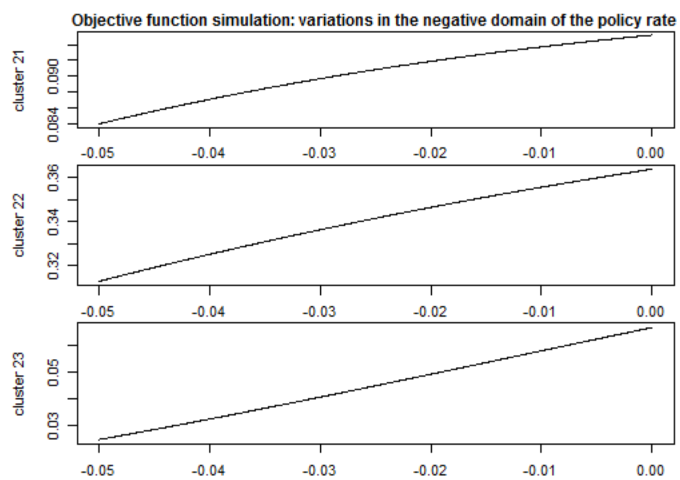


Figure 3: Optimal objective function simulation for varying negative policy rates, cluster by cluster.

The options determining the objective function loose value for negative rates and recover it when approaching the zero upper bound. In this simulation we allowed the negative rate to decrease till -5%. The shape of the curve in this negative domain for the interest rate, do not differ too much between the two options because for both of them the moneyness is impaired. In this graph we show that in the simulation we reach an upper bound for

[17]including Switzerland, with a negative rate on the longest maturity

the objective function at a zero level of the policy rate, when constraining it to the range $[-5\%, 0]$. Allowing for such a range, we perform the optimization of the objective function and we found that the optimal policy rate is always zero. This is reasonable given the definition of the objective function itself. In the real world, hopefully, the policy rate is not going so far in the negative domain. Thus, we propose the following graph to focus on a narrower range $[-1\%; 0]$.

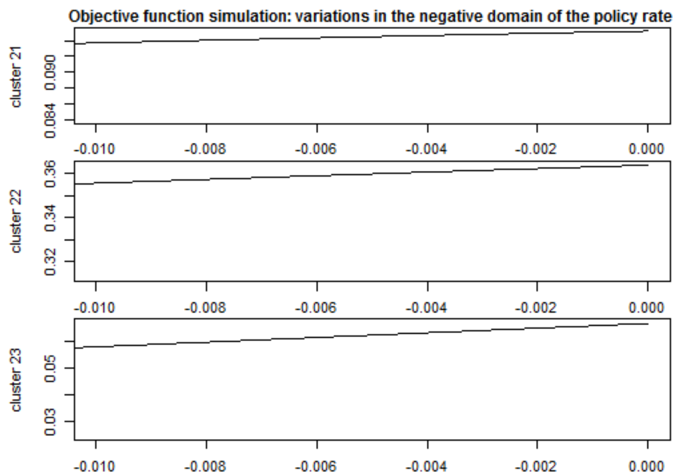


Figure 4: Optimal objective function simulation for small variations in negative policy rates, cluster by cluster.

In this case we are able appreciate the *magnitudo* of the impact such discount rate has on the option value. The curve are quite flat, thus *ceteris paribus* we can argue that the negative rates in our optimization procedure impact negatively on the objective function, but effect is not that relevant as in the case of small change in values of the other optimizing variables. We estimate that with a variation of -1% in the policy rate, the objective function decrease by 1.2% in case of cluster 22, 0.8% in case of cluster 23. In case of cluster 21, there is an almost zero variation in this second case and a decrease of 1% when the policy rate decrease by 5%. Nevertheless, we cannot forget the burden the monetary policy maker puts on the bank manager shoulders setting negative rates. Negative rates provide a strong motivation to shift the the bank's investments from safer to riskier ones. A possible extension to this model would consider the required increase in assets' and franchise value's volatility required to compensate for the negative interest rates.

B: Confidence intervals

	cluster	Lower bounds	Upper bounds	number of observations
	21	427347502	1087468847	4244
Average (Franchise Value (NPV))	**22**	1294026829	3492363319	4024
(in US$)	**23**	42450671293	59844573718	1071
	21	1828805281	3155078155	4244
Average (Assets (MV))	**22**	3531649196	7928260545	4024
(in US$)	**23**	85686878091	120446364427	1071
	21	0.1092981	0.1104778	4244
Average (Optimal leverage)	**22**	0.07247293	0.07305315	4024
	23	0.03814159	0.03974975	1071
	21	0.03989881	0.04163346	4244
Average (Optimal volatility)	**22**	0.03538589	0.03696676	4024
	23	0.02201921	0.02396731	1071
	21	0.1261484	0.1323115	4244
Average (Optimal policy rate)	**22**	0.1621953	0.1682553	4024
	23	0.1658552	0.1776327	1071

Table 4: Confidence intervals for average optimal estimates.

C: Summary statistics of the optimization figures at aggregate level and cluster by cluster

First of all we present summary statistics of our input variables: end-of-year equity market value, its monthly volatility adjusted on an annual basis, the risk-free rate, existent in the market in the time span considered, the market value of debt, calculated according to KMV model in order to account for the value that triggers the franchise value of the bank and the leverage defined as the ratio between the sum of the market value of the assets (MVA) and the franchise value (Fr)and the equity market value [18](MVE).

[18]Leverage data are in line with findings in Kalemli-Ozcan et al. (2012).

Variable\Summary statistics	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.	sd
Equity market value (in US$)	1.922e+05	4.324e+07	1.012e+08	1.050e+09	2.784e+08	2.339e+11	7522590845
Equity volatility (annualized)	0.0610	0.2553	0.3073	0.3484	0.3991	0.4234	0.1474
Risk-free rate	0.0100	0.0700	0.1600	0.1521	0.2200	0.3300	0.0863
Debt market value (in US$)	3.140e+05	3.971e+08	8.661e+08	1.812e+10	2.263e+09	2.782e+12	124251669021
Leverage $tier17\,(MVA + Fr)$	0.02849	0.07118	0.08927	0.09417	0.11500	0.15000	0.03322

Table 5: Model inputs summary statistics.

Those are the inputs used to estimate the franchise value first and consequently to proceed in our objective function maximization. We provide in the following table, the summary statistics for the results of our two-steps optimization at aggregate level: the net present value (NPV) of the franchise value, the market value (MV) of the assets, the parameters optimizing pointwise the objective function (leverage, franchise value and assets' volatility and the optimal risk-free rate).

Variable\Summary statistics	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.	sd
Franchise Value (NPV) (in US$)	0.000e+00	2.667e+06	6.914e+07	6.164e+09	2.450e+08	1.638e+12	52747617409
Assets (MV) (in US$)	0.000e+00	3.526e+08	7.621e+08	1.342e+10	1.881e+09	3.275e+12	105421665638
Optimal leverage	0.02759	0.07025	0.08802	0.09378	0.11380	0.14840	0.03182165
Optimal volatility	0.00010	0.02561	0.03807	0.04363	0.06247	0.08260	0.02508671
Optimal risk-free rate	-0.0500	0.0700	0.1600	0.1461	0.2207	0.33000	0.1012666

Table 6: Results summary statistics.

In the table below we present the output variables for our optimization cluster by cluster.

Summary statistics/ Variable	cluster	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.	sd
Optimal leverage	**21**	0.06737	0.09436	0.10490	0.10990	0.12440	0.14250	0.01960008
	22	0.05112	0.06558	0.07330	0.07276	0.08049	0.09549	0.009386614
	23	0.00437	0.02614	0.04046	0.03895	0.05134	0.07656	0.01341079
Optimal volatility	**21**	0.001042	0.027460	0.038180	0.040750	0.054170	0.075310	0.02118565
	22	0.002282	0.025110	0.034470	0.036160	0.047450	0.066650	0.01773035
	23	0.00872	0.01791	0.02175	0.02295	0.02810	0.03478	0.0072292
Optimal risk-free rate	**21**	-0.0500	0.0500	0.1463	0.1292	0.2100	0.3300	0.1023966
	22	-0.0500	0.1000	0.1712	0.1652	0.2400	0.3300	0.09803755
	23	-0.0500	0.0900	0.1800	0.1717	0.2500	0.3300	0.09821578

Table 7: Model results summary statistics, cluster by cluster.

C: Rho, Vega, derivative with respect to leverage and the joint derivatives

In this paragraph, for exposition reasons, leverage is defined as the ratio between the market value of straight debt and the market value of the assets together with the franchise value.

$$\text{ObjectiveFunction}\,(lev, sig\_A\_Fr, rf) :=$$

$$lev\;(MVA+Fr)\;\text{cdf\_normal}\left(\frac{-\left(\log\left(\frac{1}{lev}\right)+\left(rf-0.5\,sig\_A\_Fr^2\right)\,T\right)}{sig\_A\_Fr\,\sqrt{T}},0,1\right)+$$
$$-\,(MVA+Fr)\;\text{cdf\_normal}\left(\frac{-\left(\log\left(\frac{1}{lev}\right)+\left(rf+0.5\,sig\_A\_Fr^2\right)\,T\right)}{sig\_A\_Fr\,\sqrt{T}},0,1\right)+$$
$$+\,Fr\left(\text{cdf\_normal}\left(\frac{\log\left(\frac{1}{lev}\right)}{sig\_A\_Fr\,\sqrt{T}}+sig\_A\_Fr\,\sqrt{T}\,\frac{rf+0.5\,sig\_A\_Fr^2}{sig\_A\_Fr^2},0,1\right)\right)+$$
$$-\,Fr\left(lev^{2\,\frac{rf+0.5\,sig\_A\_Fr^2}{sig\_A\_Fr^2}}\;\text{cdf\_normal}\left(\frac{\log\left(lev\right)}{sig\_A\_Fr\,\sqrt{T}}+sig\_A\_Fr\,\sqrt{T}\,\frac{rf+0.5\,sig\_A\_Fr^2}{sig\_A\_Fr^2},0,1\right)\right)$$

First order derivative with respect to leverage:

$$(MVA+Fr)\left(\frac{\text{erf}\left(\frac{\log(lev)-\left(rf-0.5\,sig\_A\_Fr^2\right)\,T}{\sqrt{2}\,sig\_A\_Fr\,\sqrt{T}}\right)}{2}+\frac{1}{2}\right)-\frac{(MVA+Fr)\;e^{-\frac{\left(\log(lev)-\left(0.5\,sig\_A\_Fr^2+rf\right)\,T\right)^2}{2\,sig\_A\_Fr^2\,T}}}{\sqrt{2}\,\sqrt{\pi}\,lev\,sig\_A\_Fr\,\sqrt{T}}+$$
$$+\,\frac{(MVA+Fr)\;e^{-\frac{\left(\log(lev)-\left(rf-0.5\,sig\_A\_Fr^2\right)\,T\right)^2}{2\,sig\_A\_Fr^2\,T}}}{\sqrt{2}\,\sqrt{\pi}\,sig\_A\_Fr\,\sqrt{T}}+Fr$$

$$(-\frac{lev^{\frac{2\left(0.5\,\mathit{sig\_A\_Fr}^{2}+rf\right)}{\mathit{sig\_A\_Fr}^{2}}-1}\,e^{-\frac{\left(\frac{\left(0.5\,\mathit{sig\_A\_Fr}^{2}+rf\right)\sqrt{T}}{\mathit{sig\_A\_Fr}}+\frac{\log(lev)}{\mathit{sig\_A\_Fr}\sqrt{T}}\right)^{2}}{2}}}{\sqrt{2}\sqrt{\pi}\,\mathit{sig\_A\_Fr}\sqrt{T}}+$$

$$-\frac{e^{-\frac{\left(\frac{\left(0.5\,\mathit{sig\_A\_Fr}^{2}+rf\right)\sqrt{T}}{\mathit{sig\_A\_Fr}}-\frac{\log(lev)}{\mathit{sig\_A\_Fr}\sqrt{T}}\right)^{2}}{2}}}{\sqrt{2}\sqrt{\pi}\,lev\,\mathit{sig\_A\_Fr}\sqrt{T}}-\frac{2\,lev^{\frac{2\left(0.5\,\mathit{sig\_A\_Fr}^{2}+rf\right)}{\mathit{sig\_A\_Fr}^{2}}-1}\left(0.5\,\mathit{sig\_A\_Fr}^{2}+rf\right)\left(\frac{\operatorname{erf}\left(\frac{\frac{\left(0.5\,\mathit{sig\_A\_Fr}^{2}+rf\right)\sqrt{T}}{\mathit{sig\_A\_Fr}}+\frac{\log(lev)}{\mathit{sig\_A\_Fr}\sqrt{T}}}{\sqrt{2}}\right)}{2}+\frac{1}{2}\right)}{\mathit{sig\_A\_Fr}^{2}}$$

Vega, first order derivative with respect to volatility:

$$-\frac{(MVA+Fr)\left(-\frac{\log(lev)-\left(0.5\,\mathit{sig\_A\_Fr}^{2}+rf\right)T}{\sqrt{2}\,\mathit{sig\_A\_Fr}^{2}\sqrt{T}}-\frac{1.0\sqrt{T}}{\sqrt{2}}\right)e^{-\frac{\left(\log(lev)-\left(0.5\,\mathit{sig\_A\_Fr}^{2}+rf\right)T\right)^{2}}{2\,\mathit{sig\_A\_Fr}^{2}\,T}}}{\sqrt{\pi}}+$$

$$+\frac{lev\ (MVA+Fr)\left(\frac{1.0\sqrt{T}}{\sqrt{2}}-\frac{\log(lev)-\left(rf-0.5\,\mathit{sig\_A\_Fr}^{2}\right)T}{\sqrt{2}\,\mathit{sig\_A\_Fr}^{2}\sqrt{T}}\right)e^{-\frac{\left(\log(lev)-\left(rf-0.5\,\mathit{sig\_A\_Fr}^{2}\right)T\right)^{2}}{2\,\mathit{sig\_A\_Fr}^{2}\,T}}}{\sqrt{\pi}}+$$

$$+Fr\,(-\frac{lev^{\frac{2\left(0.5\,\mathit{sig\_A\_Fr}^{2}+rf\right)}{\mathit{sig\_A\_Fr}^{2}}}\left(-\frac{\left(0.5\,\mathit{sig\_A\_Fr}^{2}+rf\right)\sqrt{T}}{\mathit{sig\_A\_Fr}^{2}}+1.0\sqrt{T}-\frac{\log(lev)}{\mathit{sig\_A\_Fr}^{2}\sqrt{T}}\right)e^{-\frac{\left(\frac{\left(0.5\,\mathit{sig\_A\_Fr}^{2}+rf\right)\sqrt{T}}{\mathit{sig\_A\_Fr}}+\frac{\log(lev)}{\mathit{sig\_A\_Fr}\sqrt{T}}\right)^{2}}{2}}}{\sqrt{2}\sqrt{\pi}}+$$

$$\frac{\left(-\frac{\left(0.5\,\mathit{sig\_A\_Fr}^{2}+rf\right)\sqrt{T}}{\mathit{sig\_A\_Fr}^{2}}+1.0\sqrt{T}+\frac{\log(lev)}{\mathit{sig\_A\_Fr}^{2}\sqrt{T}}\right)e^{-\frac{\left(\frac{\left(0.5\,\mathit{sig\_A\_Fr}^{2}+rf\right)\sqrt{T}}{\mathit{sig\_A\_Fr}}-\frac{\log(lev)}{\mathit{sig\_A\_Fr}\sqrt{T}}\right)^{2}}{2}}}{\sqrt{2}\sqrt{\pi}}+$$

$$-\,lev^{\frac{2\left(0.5\,\mathit{sig\_A\_Fr}^{2}+rf\right)}{\mathit{sig\_A\_Fr}^{2}}}\log(lev)\left(\frac{2.0}{\mathit{sig\_A\_Fr}}-\frac{4\left(0.5\,\mathit{sig\_A\_Fr}^{2}+rf\right)}{\mathit{sig\_A\_Fr}^{3}}\right)\left(\frac{\operatorname{erf}\left(\frac{\frac{\left(0.5\,\mathit{sig\_A\_Fr}^{2}+rf\right)\sqrt{T}}{\mathit{sig\_A\_Fr}}+\frac{\log(lev)}{\mathit{sig\_A\_Fr}\sqrt{T}}}{\sqrt{2}}\right)}{2}+\frac{1}{2}\right))$$

Rho, first order derivative with respect to the policy rate:

$$\frac{(MVA+Fr)\sqrt{T}\,e^{-\frac{\left(\log(lev)-\left(0.5\,\mathit{sig\_A\_Fr}^{2}+rf\right)T\right)^{2}}{2\,\mathit{sig\_A\_Fr}^{2}\,T}}}{\sqrt{2}\sqrt{\pi}\,\mathit{sig\_A\_Fr}}-\frac{lev\ (MVA+Fr)\sqrt{T}\,e^{-\frac{\left(\log(lev)-\left(rf-0.5\,\mathit{sig\_A\_Fr}^{2}\right)T\right)^{2}}{2\,\mathit{sig\_A\_Fr}^{2}\,T}}}{\sqrt{2}\sqrt{\pi}\,\mathit{sig\_A\_Fr}}+$$

$$+Fr\,(-\frac{lev^{\frac{2\left(0.5\,\mathit{sig\_A\_Fr}^{2}+rf\right)}{\mathit{sig\_A\_Fr}^{2}}}\sqrt{T}\,e^{-\frac{\left(\frac{\left(0.5\,\mathit{sig\_A\_Fr}^{2}+rf\right)\sqrt{T}}{\mathit{sig\_A\_Fr}}+\frac{\log(lev)}{\mathit{sig\_A\_Fr}\sqrt{T}}\right)^{2}}{2}}}{\sqrt{2}\sqrt{\pi}\,\mathit{sig\_A\_Fr}}+\frac{\sqrt{T}\,e^{-\frac{\left(\frac{\left(0.5\,\mathit{sig\_A\_Fr}^{2}+rf\right)\sqrt{T}}{\mathit{sig\_A\_Fr}}-\frac{\log(lev)}{\mathit{sig\_A\_Fr}\sqrt{T}}\right)^{2}}{2}}}{\sqrt{2}\sqrt{\pi}\,\mathit{sig\_A\_Fr}}-$$

$$2\,lev^{\frac{2\left(0.5\,sig\_A\_Fr^2+rf\right)}{sig\_A\_Fr^2}}\log(lev)\left(\frac{\operatorname{erf}\left(\frac{\frac{\left(0.5\,sig\_A\_Fr^2+rf\right)\sqrt{T}}{sig\_A\_Fr}+\frac{\log(lev)}{sig\_A\_Fr\,\sqrt{T}}}{\sqrt{2}}\right)}{2}+\frac{1}{2}\right)$$
$$\overline{\qquad sig\_A\_Fr^2 \qquad})$$

First joint derivative with respect to both leverage and volatility:

$$\frac{d^{sig\_A\_Fr}}{d\,lev^{sig\_A\_Fr}}\left(-(MVA+Fr)\left(\frac{\operatorname{erf}\left(\frac{\log(lev)-\left(0.5\,sig\_A\_Fr^2+rf\right)T}{\sqrt{2}\,sig\_A\_Fr\,\sqrt{T}}\right)}{2}+\frac{1}{2}\right)+lev\,(MVA+Fr)\left(\frac{\operatorname{erf}\left(\frac{\log(lev)-\left(rf-0.5\,sig\_A\_Fr^2\right)T}{\sqrt{2}\,sig\_A\_Fr\,\sqrt{T}}\right)}{2}+\frac{1}{2}\right)\right)+$$
$$+\frac{d^{sig\_A\_Fr}}{d\,lev^{sig\_A\_Fr}}\left(Fr\left(-lev^{\frac{2\left(0.5\,sig\_A\_Fr^2+rf\right)}{sig\_A\_Fr^2}}\left(\frac{\operatorname{erf}\left(\frac{\frac{\left(0.5\,sig\_A\_Fr^2+rf\right)\sqrt{T}}{sig\_A\_Fr}+\frac{\log(lev)}{sig\_A\_Fr\,\sqrt{T}}}{\sqrt{2}}\right)}{2}+\frac{1}{2}\right)\right)\right)+$$
$$+\frac{d^{sig\_A\_Fr}}{d\,lev^{sig\_A\_Fr}}\left(Fr\left(+\frac{\operatorname{erf}\left(\frac{\frac{\left(0.5\,sig\_A\_Fr^2+rf\right)\sqrt{T}}{sig\_A\_Fr}-\frac{\log(lev)}{sig\_A\_Fr\,\sqrt{T}}}{\sqrt{2}}\right)}{2}+\frac{1}{2}\right)\right)$$

Second joint derivative with respect to both leverage and policy rate:

$$\frac{d^{rf}}{d\,lev^{rf}}\left(-(MVA+Fr)\left(\frac{\operatorname{erf}\left(\frac{\log(lev)-\left(0.5\,sig\_A\_Fr^2+rf\right)T}{\sqrt{2}\,sig\_A\_Fr\,\sqrt{T}}\right)}{2}+\frac{1}{2}\right)+lev\,(MVA+Fr)\left(\frac{\operatorname{erf}\left(\frac{\log(lev)-\left(rf-0.5\,sig\_A\_Fr^2\right)T}{\sqrt{2}\,sig\_A\_Fr\,\sqrt{T}}\right)}{2}+\frac{1}{2}\right)\right)+$$
$$+\frac{d^{rf}}{d\,lev^{rf}}\left(+Fr\left(-lev^{\frac{2\left(0.5\,sig\_A\_Fr^2+rf\right)}{sig\_A\_Fr^2}}\left(\frac{\operatorname{erf}\left(\frac{\frac{\left(0.5\,sig\_A\_Fr^2+rf\right)\sqrt{T}}{sig\_A\_Fr}+\frac{\log(lev)}{sig\_A\_Fr\,\sqrt{T}}}{\sqrt{2}}\right)}{2}+\frac{1}{2}\right)+\frac{\operatorname{erf}\left(\frac{\frac{\left(0.5\,sig\_A\_Fr^2+rf\right)\sqrt{T}}{sig\_A\_Fr}-\frac{\log(lev)}{sig\_A\_Fr\,\sqrt{T}}}{\sqrt{2}}\right)}{2}+\frac{1}{2}\right)\right)$$

Third joint derivative with respect to the policy rate and volatility:

$$\frac{d^{rf}}{d\,sig\_A\_Fr^{rf}}\left(-(MVA+Fr)\left(\frac{\operatorname{erf}\left(\frac{\log(lev)-\left(0.5\,sig\_A\_Fr^2+rf\right)T}{\sqrt{2}\,sig\_A\_Fr\,\sqrt{T}}\right)}{2}+\frac{1}{2}\right)+lev\,(MVA+Fr)\left(\frac{\operatorname{erf}\left(\frac{\log(lev)-\left(rf-0.5\,sig\_A\_Fr^2\right)T}{\sqrt{2}\,sig\_A\_Fr\,\sqrt{T}}\right)}{2}+\frac{1}{2}\right)\right)+$$

$$+\frac{d^{rf}}{d\,sig\_A\_Fr^{rf}}\left(+Fr\left(-lev^{\frac{2\left(0.5\,sig\_A\_Fr^2+rf\right)}{sig\_A\_Fr^2}}\left(\frac{\operatorname{erf}\left(\frac{\frac{\left(0.5\,sig\_A\_Fr^2+rf\right)\sqrt{T}}{sig\_A\_Fr}+\frac{\log(lev)}{sig\_A\_Fr\,\sqrt{T}}}{\sqrt{2}}\right)}{2}+\frac{1}{2}\right)\right)\right)+$$

$$+\frac{d^{rf}}{d\,sig\_A\_Fr^{rf}}\left(+Fr\left(+\frac{\operatorname{erf}\left(\frac{\frac{\left(0.5\,sig\_A\_Fr^2+rf\right)\sqrt{T}}{sig\_A\_Fr}-\frac{\log(lev)}{sig\_A\_Fr\,\sqrt{T}}}{\sqrt{2}}\right)}{2}+\frac{1}{2}\right)\right)$$

Second-order derivative with respect to leverage:

$$\frac{d^{lev}}{d\,lev^{lev}}\left(-\left(MVA+Fr\right)\left(\frac{\operatorname{erf}\left(\frac{\log(lev)-\left(0.5\,sig\_A\_Fr^2+rf\right)T}{\sqrt{2}\,sig\_A\_Fr\,\sqrt{T}}\right)}{2}+\frac{1}{2}\right)+lev\,\left(MVA+Fr\right)\left(\frac{\operatorname{erf}\left(\frac{\log(lev)-\left(rf-0.5\,sig\_A\_Fr^2\right)T}{\sqrt{2}\,sig\_A\_Fr\,\sqrt{T}}\right)}{2}+\frac{1}{2}\right)\right)+$$

$$+\frac{d^{lev}}{d\,lev^{lev}}$$

$$\left(+Fr\left(-lev^{\frac{2\left(0.5\,sig\_A\_Fr^2+rf\right)}{sig\_A\_Fr^2}}\left(\frac{\operatorname{erf}\left(\frac{\frac{\left(0.5\,sig\_A\_Fr^2+rf\right)\sqrt{T}}{sig\_A\_Fr}+\frac{\log(lev)}{sig\_A\_Fr\,\sqrt{T}}}{\sqrt{2}}\right)}{2}+\frac{1}{2}\right)+\frac{\operatorname{erf}\left(\frac{\frac{\left(0.5\,sig\_A\_Fr^2+rf\right)\sqrt{T}}{sig\_A\_Fr}-\frac{\log(lev)}{sig\_A\_Fr\,\sqrt{T}}}{\sqrt{2}}\right)}{2}+\frac{1}{2}\right)\right)$$

Vomma, second-order derivative with respect to volatility:

$$\frac{d^{sig\_A\_Fr}}{d\,sig\_A\_Fr^{sig\_A\_Fr}}\left(-\left(MVA+Fr\right)\left(\frac{\operatorname{erf}\left(\frac{\log(lev)-\left(0.5\,sig\_A\_Fr^2+rf\right)T}{\sqrt{2}\,sig\_A\_Fr\,\sqrt{T}}\right)}{2}+\frac{1}{2}\right)+lev\,\left(MVA+Fr\right)\left(\frac{\operatorname{erf}\left(\frac{\log(lev)-\left(rf-0.5\,sig\_A\_Fr^2\right)T}{\sqrt{2}\,sig\_A\_Fr\,\sqrt{T}}\right)}{2}+\frac{1}{2}\right)\right)$$

$$\frac{d^{sig\_A\_Fr}}{d\,sig\_A\_Fr^{sig\_A\_Fr}}\left(+Fr\left(-lev^{\frac{2\left(0.5\,sig\_A\_Fr^2+rf\right)}{sig\_A\_Fr^2}}\left(\frac{\operatorname{erf}\left(\frac{\frac{\left(0.5\,sig\_A\_Fr^2+rf\right)\sqrt{T}}{sig\_A\_Fr}+\frac{\log(lev)}{sig\_A\_Fr\,\sqrt{T}}}{\sqrt{2}}\right)}{2}+\frac{1}{2}\right)\right)\right)+$$

$$+\frac{d^{sig\_A\_Fr}}{d\,sig\_A\_Fr^{sig\_A\_Fr}}\left(+Fr\left(\frac{\operatorname{erf}\left(\frac{\frac{\left(0.5\,sig\_A\_Fr^2+rf\right)\sqrt{T}}{sig\_A\_Fr}-\frac{\log(lev)}{sig\_A\_Fr\,\sqrt{T}}}{\sqrt{2}}\right)}{2}+\frac{1}{2}\right)\right)$$

Second-order derivative with respect to the policy rate:

$$\frac{d^{rf}}{d\,rf^{rf}} \left(-\left(MVA+Fr\right) \left(\frac{\operatorname{erf}\left(\frac{\log(lev)-\left(0.5\, sig\_A\_Fr^{2}+rf\right)T}{\sqrt{2}\, sig\_A\_Fr\, \sqrt{T}} \right)}{2} + \frac{1}{2} \right) + lev\, \left(MVA+Fr\right) \left(\frac{\operatorname{erf}\left(\frac{\log(lev)-\left(rf-0.5\, sig\_A\_Fr^{2}\right)T}{\sqrt{2}\, sig\_A\_Fr\, \sqrt{T}} \right)}{2} + \frac{1}{2} \right) \right) +$$

$$+\frac{d^{rf}}{d\,rf^{rf}} \left(+Fr \left(-lev^{\frac{2\left(0.5\, sig\_A\_Fr^{2}+rf\right)}{sig\_A\_Fr^{2}}} \left(\frac{\operatorname{erf}\left(\frac{\frac{\left(0.5\, sig\_A\_Fr^{2}+rf\right)\sqrt{T}}{sig\_A\_Fr} + \frac{\log(lev)}{sig\_A\_Fr\,\sqrt{T}}}{\sqrt{2}} \right)}{2} + \frac{1}{2} \right) + \frac{\operatorname{erf}\left(\frac{\frac{\left(0.5\, sig\_A\_Fr^{2}+rf\right)\sqrt{T}}{sig\_A\_Fr} - \frac{\log(lev)}{sig\_A\_Fr\,\sqrt{T}}}{\sqrt{2}} \right)}{2} + \frac{1}{2} \right) \right)$$

Chapter 2

Optimal Bank Risk Appetite in a World of CoCos[19]

Abstract

We investigate the shape of risk appetite when the bank is financed also with contingent convertible bonds (CoCos). Our contribution to the existent literature is to assess risk appetite in a multi-dimensional perspective and to account for differences among banks' clusters, especially in a world with CoCos and policy rates approaching zero or negative figures. In our model, the bank objective function is given by the sum of the default put option and the down-and-out call (DOC) option, pricing the net present value of growth opportunities. The manager maximizes the market value of the bank, adjusting jointly the level of leverage, the amount of CoCos to issue, assets and franchise value volatility and the policy rate. Risk appetite is given by the first order derivatives. Our model and Basel III recommendation converge over their incentives regarding volatility and leverage. For banks with higher franchise value, it is optimal to issue an average amount of CoCos (over the market value of the assets) of 3%. A decrease in volatility accrues to the DOC option, in particular for under capitalized banks. The optimal policy rate is always higher with respect to the actual one. The optimal value for the decision variables differs among the clusters. We show that for the bank, it is always better to issue CoCos with respect to write-down bonds.

Keywords: Contingent Convertible Bonds, risk appetite, franchise value, Basel III.
JEL classification: G21, G32, G38, E52

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6 Introduction

Following the failure of Lehman Brothers, governments announced the end of the *too big to fail*. In this context, issuing loss absorbing instruments has gained increasing popularity. Between 2009 and 2015, banks issued more than USD380 billion of CoCos [20]. Regulation plays a crucial role in determining CoCos' issuance. Under Basel III, CoCo bonds are eligible as either Additional Tier 1 (AT1) or Tier 2 (T2), which are types of capital apparently preferred by banks with respect to equity to accomplish regulatory requirements, given that they are cheaper and less dilutive than issuing equity. Their introduction into the financing structure of our banks is relevant from a regulation and risk management point of view. There is no convergence in the opinions concerning those instruments mainly due to the uncertainty around their impact and the difficulties in understanding their conversion mechanisms. There are CoCos supporters, like Switzerland's FINMA and Bank of England, and opponents, such as Deutsche Bank.[21] Our analysis is driven by the concerns economists have regarding the impact of CoCos and WDs on bank risk appetite. Neel Kashkari, Minneapolis FED's President, warns against instruments like CoCos, suggesting that "..transferring risk to investors won't protect taxpayers from bailout".[22]

In this paper, we introduce CoCo bonds in the financing structure of our bank and see how the shape of the bank risk appetite changes. In our model, the manager acts in order to accomplish regulatory requirements. Focusing on capital requirements, banks, accomplishing to Basel III rule, have to fund themselves with at least 4.5% of common equity of risk-weighted assets (RWAs). The regulator allows for an extra 1.5% of Additional Tier 1 (AT1) that together with the Common Equity Tier 1 (CET1) concurs to compose the minimum level of 6% of Tier 1 capital over RWAs. Hence, in order to be compliant with this ratio, the manager has discretionary power over two variables: CoCos issuance and assets' volatility. On one side, issuing CoCos, which are eligible for AT1, the manager increases the numerator of the ratio, enlarging the Tier 1. On the other side, the manager might decrease the RWAs. RWAs are a weighted sum of banks' assets, weighted for their contribution to the total assets' volatility. Through this capital requirement ratio, the regulator provides an incentive for decreasing assets' volatility. In our model, optimizing the level of risk appetite, the manager should focus on the maximization of the bank objective function, which is given by the sum of the two options (default put option and down-and-out call (DOC) option). We assess the impact on our

[20] This amount refer to the face value of the CoCos issued, Data from Moody's Investors Service, Moody's Quarterly Rated and Tracked CoCo Monitor Database- Year End 2015.

[21] Financial Times, ECB is having second thoughts on CoCo bonds, 04.24.2016, $https : //www.ft.com/content/23d61e50 - 08a7 - 11e6 - b6d3 - 746f8e9cdd33$.

[22] The Wall Street Journal, Fed's Kashkari Says Transferring Risk to Investors Won't Protect Taxpayers From Bailout, 04.18.2016, $http : //www.wsj.com/articles/feds - kashkari - says - transferring - risk - to - investors - wont - protect - taxpayers - from - bailout - 1460997006$.

objective function of issuing CoCos, in order to enlarge Tier1 capital, for being compliant with the capital requirement. We concentrate on the situation in which the bank deteriorates over its capital ratio, as defined in Basel III, and chooses to improve it by either decreasing assets' volatility, through assets' substitution and a change in the growth opportunities' strategy, or (and) to enlarge its Tier1 capital. The objective function is given by the sum of the two options (default put and DOC), thus the objective function is increasing with respect to volatility, but the risk to breach the barrier in the DOC option bounds volatility optimal values. In this context is not always unfavourable a decrease in volatility, this is what allows us to find an optimal value for this variable. In our case, a decrease in volatility and an increase in Tier 1 Capital favour the DOC option over the default put option. In the case of assets' deterioration, the decision to issue some instruments in order to improve its capital ratio, favour the franchise value, since the distance to the franchise barrier is again increased. This happens for CoCos both before and after the conversion, because before conversion they are eligible for AT1 and after the conversion they convert to equity concurring to increase CET1. We find that both our model and Basel III recommendation converge over their incentives regarding volatility and leverage. We show that for banks with higher franchise value it is optimal to issue an average amount of CoCos over the market value of the assets and the franchise value of 3%. CoCos do not only enlarge the distance to the default barrier but also to the franchise value's one. The optimized objective function value with CoCos is always higher than the one without CoCos, this is true both at aggregate level and in the cluster analysis. On the optimal volatility side, a decrease in this figure accrues to the DOC option, in particular for under capitalized banks even if it can't be pushed too low in order to preserve the default put option. This can be obtained by changing assets' composition and growth opportunities strategies. The other optimizing decision variables are crucial in shaping risk appetite and help in understanding the differences among the clusters. We do not differentiate before conversion between write down bonds and CoCos but only afterwards. In this context, it is always optimal to have a conversion ratio greater than zero, even if loss absorbing, thus, smaller than one, for ensuring compliance with Basel III. Our contribution to the existent literature is to assess risk appetite in a multi-dimensional perspective and to account for differences among banks' clusters which are even more relevant in a world with CoCos and policy rates approaching zero or negative figures, in a Basel III friendly framework.

7 Literature review

Our paper is mainly related to three different strands of the literature. First, the building blocks are the seminal works by Black and Scholes (1973) and Merton (1973a), where the liabilities of a company are seen as an European option written on the assets of a firm. In this context, we assess

the market value of equity building on Babbel and Merrill (2005). They introduce the concept that the franchise value, together with the default put option accrue to equity holders Barone-Adesi et al. (2014) argue that the risk appetite of banks is determined by the interplay of default put option and growth opportunities, assessed with a DOC option.In this domain, we contribute to the existent literature providing a three-dimensional framework for assessing risk appetite. We supply a superior understanding of the joint impact of the three optimizing variables (leverage, assets and franchise value volatility, and policy rate) have on each other and on the bank risk appetite. Our study takes into account Brockman and Turtle (2003), who show that equity can be knocked out whenever a legally binding barrier is breached, even if we look at this work for the pricing of our franchise value.
Second, a relevant issue for our model is the impact Basel III has on our bank's definition of risk appetite. The impact of the key instruments of Basel III is widely analysed by Hugonnier and Morellec (2017), proposing a dynamic model of banking. They find that leverage requirements decrease default risk and increase growth opportunities of the bank, on the long-run, which is in line with our findings. Another key point is that, raising equity requirements make the loss to be borne by shareholders and the distance to default increases (see e.g. Admati and Hellwig (2013)), we mitigate this counter effect by considering the issue of loss absorbing CoCos with respect to equity.
Third, we refer also to the literature of hybrid capital. An interesting literature review of the basics of this instruments is given in De Spiegeleer et al. (2014). A relevant part of the literature focuses on credit spreads, e.g. Attaoui and Poncet (2015) develop the model showing that credit spread on straight debt is lower if the firm has write-down (WD) bonds in its financing structure, given the cushion function of the WDs with respect to the senior straight debt. CoCos are nearer to equity because in some states of the world they are not debt. Chen et al. (2013) show that replacing some straight debt with CoCos lowers the endogenous default barrier and therefore increases the firm's ability to mitigate a loss in asset value. A natural direction for future research is to consider the impact of wealth transfer among different categories of stakeholders, which should be relevant for governments. Roy and El-Herraoui (2016) demonstrate the complexity of designing a fair and effective bail-in regime. The regulator is mainly confronted with the choice of implementing or not the wealth transfer. If it chooses to do so, it faces the risk of requests for compensation and arbitrage behaviour in financial markets. Our results show that banks with great positive franchise value would benefit from the inclusion of CoCos in the capital structure, and together with a low volatility and an appropriate optimal discounting rate, ensuring stability, the market value of the bank would be magnified over standard financing. This findings add to the existent literature which provides controversial results with respect to those instruments. We prove that those hybrid instruments increase the bank market value, saving its growth opportunities, when they are appropriately

counterbalanced with relative low volatility values and relative high policy rates[23].

8 The model

The subject of our study is the risk appetite of a bank held by shareholders who benefit from limited liability and who bear the down-side of a potential loss together with the CoCo holders. They discount cash flows at a constant interest rate.

The structure of the balance sheet, in book values, is given as follows. The bank owns a portfolio of risky assets and liquid reserves, and is financed by insured deposits, risky debt, CoCos and equity. At this stage we do not differentiate between CoCo bonds and WDs, because we consider only loss-absorbing CoCos. On the left hand side of the balance sheet, risky assets are relative illiquid due to informational problems (see e.g. Hugonnier and Morellec (2017) and Froot and Stein (1998).

Going to market values, debt is seen as a portfolio of cash plus a short position in a put option on firm value as in Merton (1974) and equity as a call option on assets as in Black and Scholes (1973). In our model, we focus on the interplay between the default put (PUT^{def}) option and the down-and-out call (DOC) option that accrue to shareholders, pricing the net present value of growth opportunities, i.e. the franchise value.

8.1 Main assumptions and model description

We introduce the main assumptions of our model, building on the seminal work of Black and Scholes (1973) and Merton (1974), and the following intuitions of Babbel and Merrill (2005) and Barone-Adesi et al. (2014).

We operate in continuous time, with initial date $t = 0$ and terminal date $t = T$. The usual assumptions regarding standard frictions are considered: we do not contemplate transaction costs, taxes[24], costs of raising funds, limits on short sales and riskless arbitrage opportunities. Agents are risk-neutral and conflicts of interest between shareholders, managers and CoCo holders is not a topic of this paper[25]. The focus of the project is to understand how the regulator should set appropriate risk-taking incentives in a framework where the manager has to deal together with the default put option and the DOC one. Hence, in the Optimization Problem section, we propose the objective

[23] Overall the results over the policy rates are free from considerations dealing with inflation, other macroeconomic variables and a number of factors, usually taken into account by the monetary policy. In our paper we obtain the ideal optimal discount rate, ensuring stability, given that it is determined by the interplay of the two options which display opposite sensitivities with respect to the policy rate.

[24] We briefly relax this assumption, assessing directions of future development of this model in the extensions section

[25] Again, extensions are very interesting in this domain.

function to be maximized as the sum of the two options, normalized by the market value of the assets, representing at that stage a neutral scaling value. Initially, shareholders contribute the entire equity of the bank and, subsequently, consider operating a debt-equity swap at t_0. The proceeds from debt issue are invested in the assets in place and future growth opportunities that at time T are worth $A(T)$ and $Fr(T)$, respectively[26].

The default can occur only at the end of the period, T, in case liabilities exceed assets. We define the market value of the total exposure, MVA, as the sum of the value of the tangible assets and the franchise value, subsuming the future growth opportunities of the bank. The value of the total exposure at time t is given by:

$$MVA(t) = MVA(0) \exp\left(\mu_{MVA_t} - \frac{\sigma_{MVA}^2}{2}t + \sigma_{MVA} B_t\right), \tag{21}$$

where B_t is a standard Brownian motion defined on $(\Omega, \mathcal{F}, Q)$, so that, their dynamic is:

$$d\ln(MVA(t)) = \left(\mu_{MVA_t} - \frac{\sigma_{MVA}^2}{2}t\right) dt + \sigma_{MVA} dB_t, \tag{22}$$

where the drift, μ_t, is time-varying and σ is constant and both are referred to the sum of the tangible value of the assets and the franchise value. For simplicity, we fix the risk-free rate and dividend issues equal to zero[27]. Similar to Babbel and Merril (2005) and Barone-Adesi, Farkas and Medina (2014), we split the value of the bank into three components. First, considering the limited liability, the market value of the equity of our bank is a call option on the assets:

$$E(T) := max(A(T) - L), \tag{23}$$

where A is the value of the banks'assets and L the face value of the liabilities. Second, let's split the value of equity into the following two components:

$$E(T) := X(T) + Put^{def}(T), \tag{24}$$

where $X(T) := A(T) - L$ is the net tangible value of the bank, without considering the limited liability, which is represented through the default put option. Third, we allow the bank to be able

[26] We consider as future growth opportunities not only the potential increase in credits, but all the chances the bank has to open new lines of business, to enter new markets and more in general all the potential results due to research and development.

[27] In the next subsections, we relax the assumption regarding the policy rate, allowing it to be different from zero, instead the assumption concerning the dividend yield is a topic considered in the extension as an interesting variable to take into account when comparing an issue of CoCos issue with an equity one.

to invest in value creating opportunities at time T, through the introduction of the franchise value ($Fr(T)$). Hence,

$$E(T) := X(T) + Put^{def}(T) + Fr(T). \tag{25}$$

Future growth opportunities materialize only at the end of the period, T, being $Fr\,(T)$ but the franchise, which is the net present value of future growth opportunities value might vanish previously, as soon as the liabilities exceeds the asset value in $0 \leq t \leq T$, that is when

$$\tau_{Fr=0} = inf\left\{t \geq 0 : A\,(t) \leq FV^{SD} + Dep\right\}, \tag{26}$$

where, A are the assets in place, FV^{SD} the face value of the straight debt and Dep the value of deposits. Our franchise value barrier is higher with respect to Demsetz et al. (1996) or Jones et al. (2011), because in their model this value is lost in case of bankruptcy. In our model, the ability of the firm to engage in new projects, leading to growth opportunities, may be impaired if it is perceived that the bank is experiencing a weak financial position. Some further assumptions have to be considered, because in this framework, CoCo bonds are introduced in the financing structure. At $t = 0$, the bank issues also CoCo bonds eligible for Additional Tier 1 (AT1), with infinite maturity, conforming to Basel III regulation. In contrast to straight debt, that is a zero coupon, CoCo bonds pay a coupon $c^{CoCo} > 0$ and conversion from debt to equity is triggered when the value of the firm's assets fall below an exogenously specified threshold V_{Conv}. This trigger is set larger than the face value of the standard debt, so that $FV^{SD} < V_{Conv} < A\,(t)$. Consequently, CoCos conversion occurs at

$$\tau_{Conv} = inf\left\{t \geq 0 : A\,(t) \leq V_{Conv}\right\}.$$

This implies that the optimal bankruptcy level does not depend on the conversion trigger, because it is a "post-conversion" threshold. This is relevant especially in case bankruptcy is determined endogenously, which is not the case in this model, since we rely on Basel recommendations[28]. In our model, CoCos could qualify only as AT1, under Basel III, as explained by Avdjiev et al. (2013), so that they operate in favour to franchise value as explained in the next paragraph. It means that the CoCos' threshold is higher with respect to the franchise barrier, because we consider that banks having converted CoCos still have some growth opportunities at stake. Both CoCos and franchise threshold are obviously higher with respect to bankruptcy one. The overall value for CoCo's holder is given by the sum of the face value plus the coupon payment, $c^{CoCo} > 0$, if the conversion is

[28] It would be considered in an extension in order to assess whether bank's choice would be aligned with regulation or would behave differently.

not taking place before T and the fraction of shares, $\triangle E_{\tau_{Conv}}$, CoCo holders receive, becoming shareholders in case of conversion, as identified in Alvemar and Ericson (2012):

$$V_T^{CoCo} = FV^{CoCo}1_{\{\tau_{Conv}>T\}} + c^{CoCo}1_{\{t<\tau_{Conv}\}} + \triangle E_{\tau_{Conv}}1_{\{\tau_{Conv}\leq T\}}.$$

This relation is evaluated at time T, except for the CoCos' coupon, that is due until conversion takes place. At conversion in presence of CoCos, shareholders receive a fraction of shares $\triangle E_{\tau_{Conv}}$, so that the value of the overall equity increases by such an amount. In our model, we focus only on loss-absorbing CoCos, meaning that delta is between zero and one ($0 < \triangle < 1$). When it is exactly equal to zero, the banks issues a write-down bonds, that is a contingent convertible bond converting to zero in case of a trigger event. Being in the context of banks that could consider a CoCos' issuance, we focus on Tier 1, rather than the end of the period equity market value. Thus, the end of the period Tier 1, for any $t = 0, ..., T$ is:

$$Tier1(T)=\begin{cases} A(T)-Dep-FV^{SD}(T)+Fr(T)+Put^{def}(T) & if \quad MVA(t)\geq(SD(t)+Dep(t)) \\ (SD(T)+Dep(T))-A(T) & if \quad MVA(t)<(L(t)+D(t)) \end{cases}. \tag{27}$$

We assume that the bank can default only at time $t = T$, but the franchise value might vanish before, as soon as the liabilities exceed the assets at any time between $t = 0$ and $t = T$. From this specification, we can easily understand that the main variables that determine the Tier 1 at the end of the period are the franchise value and the default put option. Hence, we define the objective function at time t, as the sum of the down-and-out call option and the default put option. We present the optimization procedure in the next sections.

In the expressions above, we show that the franchise value come to fruition in case the tangible value of the assets do not fall below the value of the total liabilities, considering also deposits. Furthermore, in case the franchise value is positive we have a first intuition regarding the superiority of the DOC option over the default put one, since the put option would be out of the money. Thus the shareholders do not exercise the put option and its present value would still be given by the option price that can be potentially exercised in the future. The opposite is true when the tangible value of the assets is eroded.

8.2 Pricing the default option in presence of CoCos

Bank shareholders are long on the default option, which the manager has to maximize acting on the behalf of the shareholders, as in Barone-Adesi et al. (2014). We propose a slight modification in the pricing formula of the default put option. We introduce also the franchise value Fr as underlying together with the standard market value of the assets MVA. This is necessary in order to prevent potential arbitrage opportunities, that could arise otherwise, buying the bank and selling short the tangible assets and the franchise value, if this last one would not be considered. Another major difference with standard literature deals with volatility which remains the volatility of the underlying, but in our case the underlying is jointly given by the market value of the assets and the franchise value(σ_{MVA}). Furthermore, introducing the CoCos in the financing, impairs the power of straight debt in stimulating the shareholders to opt for default. We account for them in leverage, i.e. lev, which is given by the ratio between the $Tier1$ and the sum of the market value of the assets and franchise value, following Basel III definition, which impose the ratio to be evaluated over the total exposure. In this case, CoCos accrue to the $Tier1$, being eligible for $AT1$.The strike price is the market value for straight debt and deposits, $MV^{SD} + Dep$ [29]. T is the time to maturity and rf is the policy rate. Our pricing for the default put option is given by:

$$
\begin{aligned}
Put^{def}\left(lev, \sigma_{MVA}, rf\right) \quad &= \left(MV^{SD} + Dep\right)\Phi\left(-d_2\right) + \\
&\quad \left(-\left(MVA\right)\Phi\left(-d_1\right)\right), \\
&\quad \text{with } \{\tau_{Fr=0} > T\}, \\
where \quad & d_1 = \left(\frac{ln\left(\frac{1}{1-lev}\right) + \left(rf + \frac{\sigma^2_{MVA}}{2}\right)T}{\sigma_{MVA}\sqrt{T}}\right), \\
lev = \left(\frac{Tier1}{MVA}\right), \quad & d_2 = d_1 - \sigma_{MVA}\sqrt{T}, \Phi - standardNormal
\end{aligned}
\tag{28}
$$

We consider without loss of generality Φ the standard Normal. In absence of growth opportunities and CoCos, the pricing formula goes back to the standard one. At a first glance, the standard greeks for this option are given as follows:

$$
\begin{aligned}
Sensitivity \quad to \quad leverage \quad &: \left[\frac{\delta Put^{def}_{i,t}}{\delta lev_{i,t}}\right] < 0 \\
Sensitivity \quad to \quad volatility \quad &: \left[\frac{\delta Put^{def}_{i,t}}{\delta \sigma_{MVA_{i,t}}}\right] > 0 \\
Sensitivity \quad to \quad policy-rate \quad &: \left[\frac{\delta Put^{def}_{i,t}}{\delta rf_{i,t}}\right] < 0
\end{aligned}
\tag{29}
$$

[29] We proxy the market value for the straight debt with the KMV model (KMV corporation).

The default put option is increasing in volatility and decreasing in leverage, defined as above, and the policy rate. This option push the bank manager to adopt a risk-taking policy, but this is mitigated, with respect to standard case, by the presence of the franchise value and CoCos, when we control also for volatility and the policy rate, as we show in the results section.

8.3 Pricing the DOC option, in presence of non-observable underlying and CoCos

We refer to the portfolio of growth opportunities at time T as $F(T)$. Before maturity, the expected value of $Fr(T)$ is not observable and embedded in the market value of the bank. At the beginning of the period, before observing the $Fr(T)$, we can price this portfolio of growth opportunities in the option framework (see for the basic financing the one proposed in Barone-Adesi et al. (2014)). The DOC is a down and out call option, with a pricing formula whose underlying is the franchise value net of investment cost in the case the bank does not opt for default. Since investment costs are already considered in the franchise value, the strike price for this option is set to zero. The barrier is given by the sum of the market value of standard senior debt and deposits. This option is priced in an European framework given that the franchise value comes to fruition only at maturity[30], but it is path dependent. In case the barrier is breached before maturity the option expires and the franchise value is driven immediately to zero. Taking into account the CoCos in the financing framework, we notice that with respect to a context with standard financing the distance to the barrier for franchise value is enlarged, both before and after conversion. This is true because before conversion CoCos accrue to $Tier1$ being eligible for $AT1$ and after conversion they become equity or nothing (depending on the conversion ratio), accruing to $Tier1$ directly being part of $CET1$. The pricing is given as follows:

$$\begin{aligned} DOC\left(lev, \sigma_{MVA}, rf\right) \quad & = Fr\left[\Phi\left(v_1\right) + \right. \\ & \left. - \left(1 - lev\right)^{2\lambda} \Phi\left(y_1\right)\right] \\ & with \left\{\tau_{Fr=0} > T\right\}, \\ where \quad & \lambda = \frac{rf + \frac{\sigma_{MVA}^2}{2}}{\sigma_{MVA}^2} \end{aligned} \tag{30}$$

$$v_1 = \frac{ln\left(\frac{1}{1-lev}\right)}{\sigma_{MVA}\sqrt{T}} + \lambda\sigma_{MVA}\sqrt{T}, \quad y_1 = \frac{ln(1-lev)}{\sigma_{MVA}\sqrt{T}} + \lambda\sigma_{MVA}\sqrt{T}$$

[30] That is equivalent to say that we can exercise it only at maturity.

This pricing formula is not applicable directly in an empirical context because the franchise value, as well as the market value of the assets and their volatility are not directly observable in the market, but we provide our model in order to estimate them in a framework considering the two options presented. We provide the standard greeks also for this option, in the context of the presence of CoCos in the financing structure of the bank which are considered in the leverage variable. Accordingly to the existent literature, the DOC option is increasing in volatility and in the policy rate. With respect to volatility we show in the empirical application that both at aggregate level and in the cluster analysis, we relation is increasing but bounded. Hence, the numerical optimization procedure, we propose in the following sections, reach the optimum in presence of relative low values for volatility. This happens because of the higher probability, in case of relative higher value of this variable, to breach the barrier and thus to set to zero the value of the franchise value. The "greek" with respect to leverage is non standard in the literature. We find it to be positive for the DOC option because, given the Basel III definition of leverage, a greater value of leverage increase the distance to the franchise value barrier. The results are given below:

$$\begin{aligned} &Sensitivity \quad to \quad leverage &: \quad &\left[\frac{\delta DOC_{i,t}}{\delta lev_{i,t}}\right] > 0 \\ &Sensitivity \quad to \quad volatility &: \quad &\left[\frac{\delta DOC_{i,t}}{\delta \sigma_{MVA_{i,t}}}\right] \lesseqgtr 0 \\ &Sensitivity \quad to \quad policy-rate &: \quad &\left[\frac{\delta DOC_{i,t}}{\delta rf_{i,t}}\right] > 0 \end{aligned} \tag{31}$$

The default put option is increasing in volatility and decreasing in leverage, defined as above, and the policy rate. We show in the following sections when the DOC prevails over the default put one in determing the shape of the objective function and consequently the one of risk appetite.

9 The optimization problem for risk appetite

In standard literature, we commonly refer to risk appetite as an assessment of the riskiness of the assets in the bank portfolio. The regulator, through the capital ratio proposed in Basel III, considers also the joint effect of volatility together with the amount of $Tier1$ at stake, that we can translate in our model into our leverage variable. We propose to go a step further, assessing the joint impact of these first two variables together with an ideal policy rate[31] which is determined by the interplay of the two options, driving our objective function. This rate is an ideal candidate ensuring stability between the opposite forces a bank decision maker has to face when shaping risk appetite. In our

[31] In the paper we refer to the policy rate or to the discount rate indiscriminately.

model we concentrate on the maximization of the market value of equity of the bank. From the relations exposed above, we know that the key determinants of the market value are the default put and the DOC option. The two options provide different incentives concerning the variables taken into account for understanding our three-dimensional risk appetite, which are leverage, assets and franchise value volatility and the policy rate. Following our reasoning, we define the objective function ($O.f.$) as the sum of the two options:

$$O.f._{i,t} := DOC\left(lev_{i,t}, \sigma_{MVA_{i,t}}, rf_{i,t}\right) + PUT^{def}\left(lev_{i,t}, \sigma_{MVA_{i,t}}, rf_{i,t}\right). \tag{32}$$

We propose this definition for the objective function because we propose a specification of the market value of the bank which is determined by the sum of the two options and is consequently driven by our decision variables (leverage, volatility and the policy rate)[32]. The bank manager has decision power over the first two variables, instead the we examine only the impact of an ideal candidate for the third one. The regulator on her side, should take into account this perspective in order to better understand banks' strategy and to set the appropriate incentives in its regulation framework. The risk appetite is determined in the optimization problem we present in Section 2.2 and is given by the first order derivatives and the determinant of the Hessian matrix.

The optimization problem is twofold. In the first step we estimate the franchise value and the market value of the assets that are not observable in the market, but are embedded in the equity market value. Those elements are necessary inputs to perform the second optimization, where we look for the optimal level of leverage, and consequently the optimal amount of CoCos to issue, the assets and franchise value volatility and the policy rate that simultaneously optimize the objective function.

9.1 First step ingredients

In the first step, the goal is to estimate the unobservable franchise value and the market value of the assets that are embedded in the equity market value. By put-call parity, we establish this first system of equations:

We solve the following system of equation simultaneously:

[32] In the section "Main assumptions and model description" and in the "First step ingredients" of the optimization problem we display how we determine the market value of the equity and thus the motivation of our objective function.

$$\begin{cases} MVE_{i,t}= & (A_{i,t}-(MV^{SD}+Dep)_{i,t}+DOC_{i,t}+Put^{def}_{i,t}), \\ \sigma_{MVE_{i,t}}MVE_{i,t}= & \sigma_{MVA_{i,t}}(MVA_{i,t})\Phi(d_{1i,t}), \\ where & d_{1i,t}=\left(\frac{ln\left(\frac{MVA_{i,t}}{(MV^{SD}+Dep)_{i,t}}\right)+\left(rf_{i,t}+\frac{\sigma^2_{MVA_{i,t}}}{2}\right)T}{\sigma_{MVA_{i,t}}\sqrt{T}}\right). \end{cases} \tag{33}$$

In the first equation, $\left(MV^{SD}+Dep\right)$ is the market value of the sum of the market value of the straight debt and deposits, considering in this first element the value of the total amount of both short term and long term liabilities. This extension of the Merton specification allows us to consider the franchise value both at the underlying and implied volatility level. We base our analysis on the intuition that the equity market value incorporates the information regarding both the assets market value and the franchise value, consequently the implied volatility estimated in this model refers to the one considering both the assets and the franchise value. The manager performs her analysis at the beginning of the period, when the franchise value is considerable only through the pricing relation of the DOC option[33]. We solve this problem through the non-linear least squares criterion function, for each bank at any time t on the whole time span considered. We minimize the distance between the data concerning the equity market value and the model extended accommodating for both the default put option and the DOC one. We perform a step by step optimization for $\Theta_{i,t} := Fr_{i,t}, A_{i,t}, \sigma_{MVA_{i,t}}$, building on the Bellman's Principle of Optimality (Bellman (1952)), applied also in Merton (1973b). We build on the following error function:

$$\begin{cases} e_{1,i,t}= & MVE_{i,t}-(A_{i,t}-(MV^{SD}+Dep)_{i,t}+DOC(\Theta_{i,t})+Put^{def}(\Theta_{i,t})), \\ e_{2,i,t}= & \sigma_{MVE_{i,t}}MVE_{i,t}-\sigma_{MVA_{i,t}}MVA_{i,t}\Phi(d_{1i,t}), \end{cases} \tag{34}$$

where $\{i\}_1^n$ is the bank identificator and $\{t\}_1^m$ the year considered. The non linear least square function is the following:

$$\Theta^*_{i,t} = \underset{(\Theta_{i,t})}{arg\min} \quad \textstyle\sum_{j,i,t=1}^{2,n,m}\left[e^2_{j,i,t}\right] \tag{35}$$

where the solution value is $\Theta^*_{i,t}$, which is the non-linear least squares estimators, optimizing the sum of the squared deviations. Hence, the needed inputs to perform the subsequent optimization[34]. At this step, empirically, we proceed in our first clustering distinguishing among banks with franchise value and without.

[33] Given that the franchise value Fr comes to fruition only at time T.

[34] As we explain in the following step, we perform the optimization at each time step t, following Bellman (1952) and Merton (1973b), in order to allow the franchise value of the bank to come to fruition at time T.

9.2 Second step

This is the central step of our optimization procedure. The key determinants of our objective function are leverage, assets and franchise value volatility and the policy rate. Thus, it would be simplistic to focus only on one dimension when considering the risk appetite of a bank. Furthermore, we will see that empirically there are major differences among banks' clusters. Hence a flat regulation, "one size fits all", is not convenient for neither the regulator nor the bank itself. In addition, we underline that the policy rate have an impact in the pricing of the two options at stake, even if the *magnitudo* of this variable is relatively small compared to the first two[35]. The strength of this paper is to consider these three dimensions and to derive a three dimensional risk appetite definition. Introducing the franchise value, we go beyond standard literature and regulation. In our model, the franchise value, gives the shape to the objective function together with the well known default put option[36]. The third key element for understanding the following relation is given by the bank's financing, including also the CoCo bonds. Accordingly to this perspective, the manager has to optimize the objective function of the bank, modifying its exposure to risky assets and changing (even if only in part, given the nature of a portfolio of growth opportunity that materializes only at T) the strategy concerning the growth opportunity portfolio(short-term - being a stepwise optimization)and adjusting bank's leverage at time zero, operating always for allowing the franchise value to come to fruition at time T. The shape of risk appetite is assessed through the determinant of the Hessian matrix in a three-dimensional perspective. We propose a volatility-driven risk appetite, as well as a leverage-driven one and a policy rate-driven one. The cluster analysis adds information, helping in the understanding of which option determines the shape of risk appetite. This element could be crucial for a more efficient regulation, which for the while differentiate only between systemically important financial institution and the rest of the banks. A priori, the sensitivity analysis of each option's value with respect to the variables at stake and we reported those results in the previous section. Overall, what is the impact of each variable marginal changes on the whole objective function in presence of franchise value and CoCos? In our framework, there is not a monetary policy maker contribution, but we assess the optimal policy rate for stability. Thus, we analyse the sensitivity of the objective function with respect to the policy rate (policy rate-driven risk appetite), setting *rho* equal to zero. The decision variables over which, instead, the manager has discretionary power at time zero are volatility and leverage. On the bank manager side, the shape of risk appetite is determined evaluating at zero both *vega* (volatility-driven risk appetite) and the first order derivative with respect to the leverage (leverage-

[35] In the empirical analysis we show sensitivity results concerning this intuition. An extension to this work would consider a quantification of the *magnitudo* each variable over the objective function.

[36] In our model the default put option is peculiar having as underlying the franchise value as well and with the presence of the CoCos in the financing of the bank.

driven risk appetite) [37]. The optimization variables are the leverage, the assets and franchise value volatility and the policy rate, so our theta in this case is: $\Theta_{i,t} := \left(lev_{i,t}, \sigma_{MVA_{i,t}}, rf_{i,t}\right)$. When the franchise value is available, the optimization problem is:

$$\Theta^*_{i,t} = \underset{\Theta_{i,t}}{arg\max} \quad [Of_{i,t}] \tag{36}$$

In this framework our three-dimensional risk appetite (R.A.) is given by:

$$\begin{aligned}
leverage - driven \quad & R.A.: \quad \left[\frac{\delta Of_{i,t}}{\delta lev_{i,t}}\right] = 0 \,|_{\sigma^*_{MVA_{i,t}}, rf^*_{i,t}} \\
volatility - driven \quad & R.A.: \quad \left[\frac{\delta Of_{i,t}}{\delta \sigma_{MVA_{i,t}}}\right] = 0 \,|_{lev^*_{i,t}, rf^*_{i,t}} \\
policy - rate - driven \quad & R.A.: \quad \left[\frac{\delta Of_{i,t}}{\delta rf_{i,t}}\right] = 0 \,|_{lev^*_{i,t}, \sigma^*_{MVA_{i,t}}}
\end{aligned} \tag{37}$$

We perform a simultaneous optimization over the three variables, hence we express the first order partial derivatives of each single variable given the optimal figures of the other two. We accommodate for a joint optimization, which helps us to go beyond the single impact of each variable, stressing the importance of taking into account the joint effect of them over the objective function. We define our optimal value of CoCos to issue per bank per year as:

$$CoCo_{i,t} := \left(lev^*_{i,t} - lev^{act}_{i,t}\right), \tag{38}$$

where $lev^{act}_{i,t}$ is the actual level of leverage in Basel terms and $lev^*_{i,t}$ is the optimal level derived in the optimization process, when the optimal level is above the actual one, otherwise we do not have an issue of CoCos. This is straightforward because the optimal leverage if greater than the actual one enlarges the $Tier1$ and this can be done either through an issue of equity, which is never optimal (we discuss this in the extensions), or through an issue of CoCos eligible for $AT1$, with a loss absorbing conversion ratio, i.e. a conversion ratio (that we call delta) smaller than one. If the conversion ratio is equal to zero, the CoCo bonds is a write down bond converting to zero, otherwise it is a proper CoCo bond. Numerically, we use the methodology developed by Byrd et al. (1995) allowing us to give lower and upper bounds for each variable (box constraints), whose initial value must satisfy the constraint. This uses a limited-memory modification of the BFGS quasi-Newton method (Broyden (1970); Fletcher (1970); Goldfarb (1970); Shanno (1970)) and in our procedure, the algorithm always

[37] In standard literature, it does not exist a "greek letter" identifying the sensitivity of an option price with respect to leverage.

achieve the finite convergence.

We perform a pointwise optimization since we are interested in the optimal values of the parameters for each bank on the whole time span. Our objective function is given by the sum of our two options, but we prefer to assess its shape considering the market value of the assets as our normalizing variable. In this optimization step, the market value of the assets is an input derived in the first step, thus our variables are totally neutral to this scaling figure. In this context, the optimal objective function do not have theoretical bounds, but we focus on $0 \leq O.f._{i,t} \leq 1$, since it is hard to find empirically a bank having the sum of the two options greater than the market value of the assets (our normalizing quantity).

9.3 A sensitivity analysis of the objective function

In the literature, we find plenty of studies concerning the greeks of the options we consider in our model. Nevertheless, given the specification we give to the DOC option and the introduction in the leverage figure of the CoCos, we give in the appendix the derivation of both the first order derivatives and the components of the Hessian matrix. For a better understanding of the empirical results of our model, we perform in the subsequent appendix a simulation for a better understanding of the optimal solutions of our objective function. The simulation is performed letting one variable free and setting the other two on average cluster input values. Given our definition of leverage, i.e. the ratio between $Tier1$ and the market value of the assets and franchise value, the sign of the first order derivative of the objective function with respect to leverage is positive if we consider only the case where the franchise value is positive. This is true because in this case the DOC option dominates over the default put one and this is favoured with a higher distance to the barrier for the franchise value. If we consider the policy rate, the two options display different sensitivities and the greeks display opposite signs. For these two variables, it is straightforward to find an optimum for our specification of objective function. This is not so obvious for volatility. At a first glance, considering an option framework, it is demanding to perceive that we can find an optimal solution for our volatility variable. On one side, the default put option is strictly increasing in volatility, but on the other side, the down-and-out call option is increasing in volatility for relative small values, but it is decreasing for larger values of this variable. This is the case because for relative higher values of volatility, there is an increasing probability to touch the barrier of our DOC, thus to drop to zero its value. This is the reason why we find relative low optimal volatility values. Summing up, for relative lower values both the options are increasing in volatility, but for relative higher values, the DOC option prevails over the default put one and the whole objective function exhibits a decreasing shape. Hence, we can find an optimal solution also for volatility.

10 Results

We implement our model on a dataset consisting of 1436 US banks, whose sample period is 1980-2014 . This is of particular interest because no US banks issued CoCos up to 2014, thus we show for which banks it would have been optimal to issue CoCos and for which not and the impact this choice, together with the others decision variables, have on the franchise value and on the objective function of the banks assessed. Balance sheet items are taken from COMPUSTAT and considered on an annual basis. Market prices from the Center for Research in Security Prices (CRSP). Price data are taken on a monthly basis to accommodate the constant volatility hypothesis. We perform our optimizations with several initial values in order to check we have results numerically stable.

10.1 Bridging our model to Basel III recommendations

Our optimization procedure leads us to the following results. First, in Figure 5 we show that the optimal volatility is below the actual embedded one. We obtain the assets' and franchise's volatility from the second step of our optimization procedure. The distance between the two figures enlarges in the years after 2008 relative to the rest of the time series. The rational for decreasing volatility following the incentive of the capital ratio (Tier1 over RWAs), where we proxy $RWAs$ for assets' and franchise's value volatility, is also motivated by our model results. We find that optimal volatility is always smaller than the volatility embedded in the market value. We demonstrate that the objective function, being determined by two options, is an increasing function of volatility, but optimal values are bounded in a relative low environment. This is important in order to preserve the franchise value which can be destroyed with high volatility given the higher probability of hitting the franchise barrier.

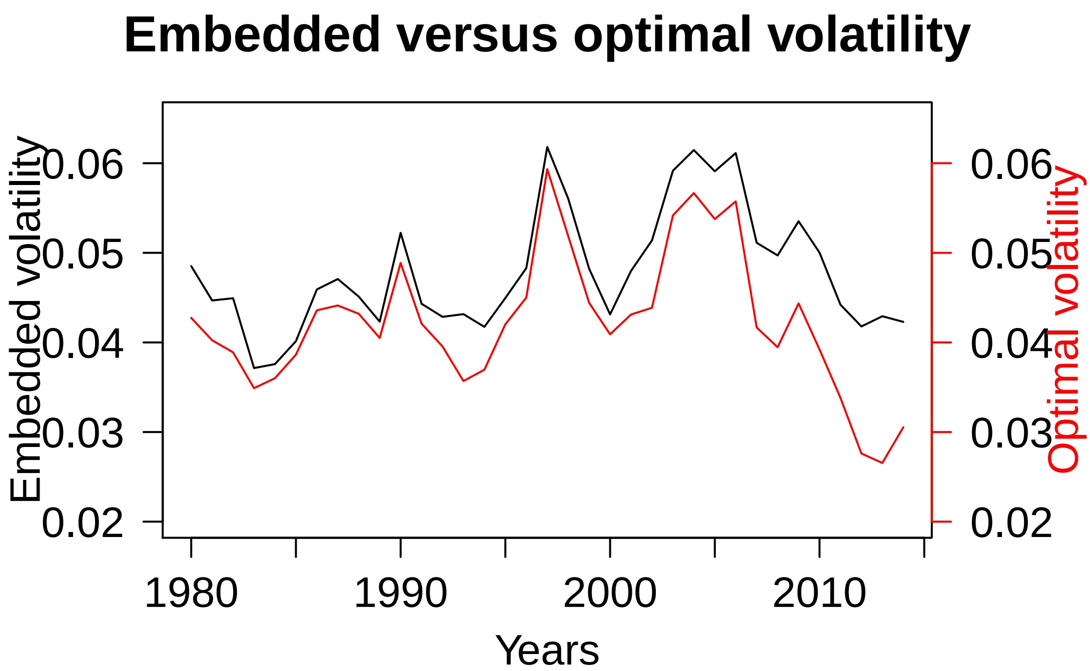


Figure 5: Embedded versus optimal volatility.

Second, we show that optimal leverage is above the actual one. We can see that the distance between the actual level and the optimal one increases substantially when the crisis of 2008 exploded. The leverage is expressed in Basel regulation terms, meaning that at the numerator we have $Tier1$ which has to be increased following both our model, considering the two options, and Basel rule. The enlargement of the $Tier1$ promotes the DOC option and helps maintaining the franchise value also during bad times, where in general we might expect an erosion of the growth opportunities of the bank. The regulation do not consider the franchise value and banks growth but in this case gives an aligned incentive. We have two ways for increasing $Tier1$. From one side, the bank can issue equity; on the other side it might opt for issuing hybrid capital eligible for $AT1$, i.e. CoCo bonds with a loss absorbing conversion ratio. We consider in the extension paragraph some pitfalls related to the issue of equity in comparison to the issue of CoCos. Issuing equity should not be optimal for both the pecking order theory, which is even more true during bad times, and for the consequences directly related to our model, dealing with discretionary coupon savings, which is greater with respect to discretionary dividends savings and the opportunity given by the tax shield. This last element is one of the main reasons why in the US banks do not issue CoCos even if we show that it should be optimal. On the other side, the fiscal regulation in Europe allow for tax shield also of this hybrid capital, thus we can see empirically an increasing market for CoCos after LLOYDs 2009 first issue. The following graph, Figure 6, represents the amount of leverage of the banks considered in our sample and is given by the ratio between $Tier1$ and the total exposure. In our model we consider as total exposure the franchise value together with the assets. This element is one of the characteristic of our model. Thus, considering also the franchise value, the banks assessed present a median yearly leverage well above Basel III recommendation (3%).

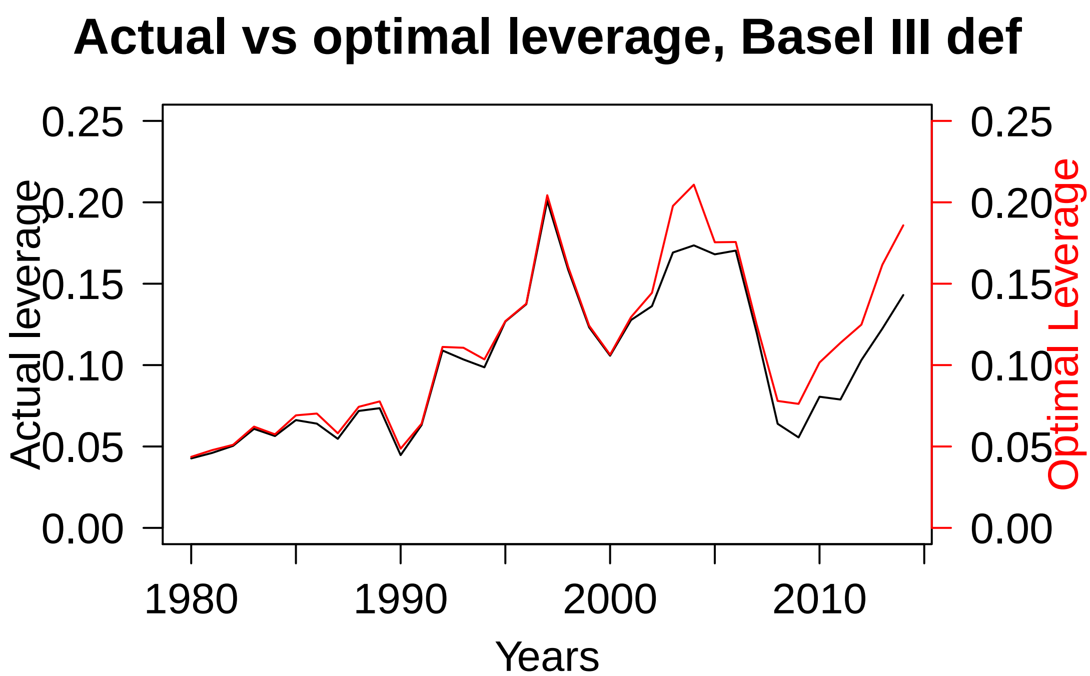


Figure 6: Actual versus optimal leverage in Basel III terms.

We conclude that the results of our model considering the decision variables over which the manager has discretionary power are in line with Basel III recommendations concerning both the leverage ratio and the capital ratio. We further propose how to understand banks potential optimal quantities of CoCos to issue and the impact an ideal optimal policy rate has over the banks' objective function.

10.2 Empirical results at aggregate level

In this subsection we present the main results at aggregate level. In Figure 7, we show the optimal average percentage amount of CoCos to issue per year. We obtain this figure as an output of our optimization procedure and it is expressed in percentage terms with respect to the market value of the assets and franchise value. Interestingly, we find a sharp increase in the last period assessed from 2007 on, starting a year before the boost of the financial crisis of 2008, leading to Basel III.

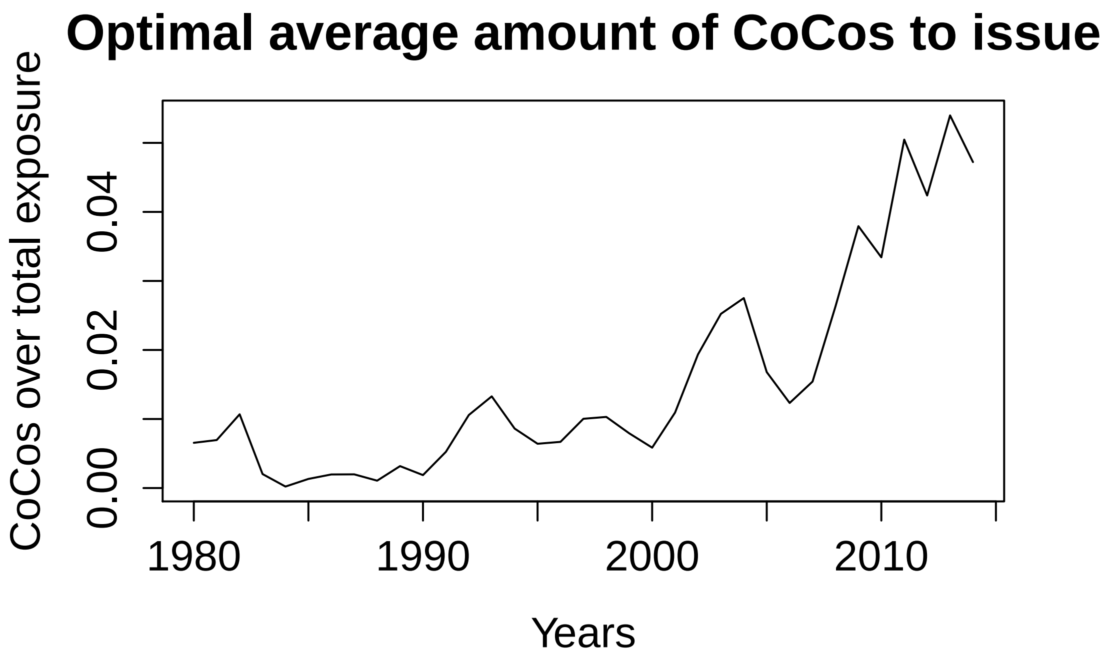


Figure 7: Optimal amount of CoCos over the market value of the assets and franchise value to issue per year.

From now on, we do consider only banks with franchise value, $cluster2$, instead it is not interesting for our model to consider the banks without it. Indeed, $cluster1$ comprehends the banks for which the franchise value is equal to zero and, in this case, the only determinant in the optimization procedure is the default put option. We further distinguish between other two sub clusters, the first one where it is optimal to issue CoCos, $cluster2a$, and the one for which is not optimal to issue CoCos, $cluster2b$. In the appendix, we display the summary statistics of the key figures for both the clusters in table 11 and table 12 as well as the related confidence intervals for the optimized variables. In Figure 8, we show a comparison of the objective function with and without CoCos. The objective function of the cluster for which is optimal to issue CoCos is always above the one of the cluster for which is not optimal to issue further hybrid capital enlarging $Tier1$. For both the clusters, the franchise value is greater than zero, thus the default put option incentives are counterbalanced by the DOC option. On one side, the sub sample for which is not optimal to issue CoCos, i.e. $cluster2b$, has relative lower optimal volatility and higher policy rate, this two factors are even more depressing the default put option, which pull down the whole optimized objective function. On the other side, the sub sample for which is optimal to issue CoCos, i.e. $cluster2a$, promotes heavily the DOC option, but thanks to a relative higher volatility and lower policy rate it maintains also the default put option value. Overall, the optimal amount of CoCos to issue is not that large with respect to total leverage and this is consistent with regulators' recommendation.

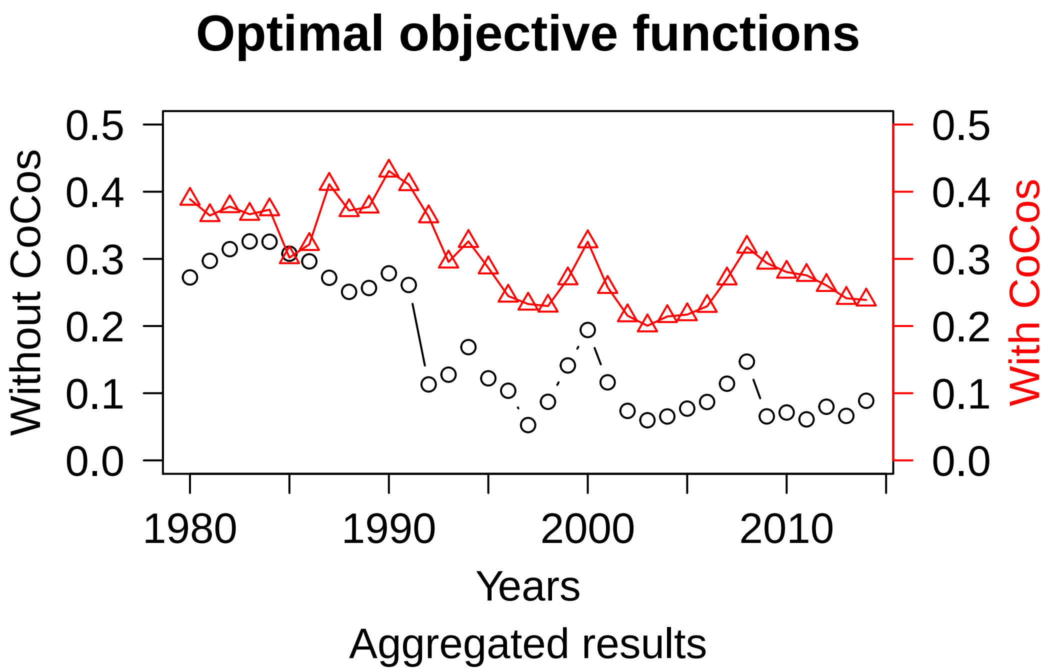


Figure 8: A comparison of the objective function with and without CoCos.

However, the DOC option is the one which drives upwards the objective function, sustained by the higher optimal leverage, which is enlarged by the CoCos issued.

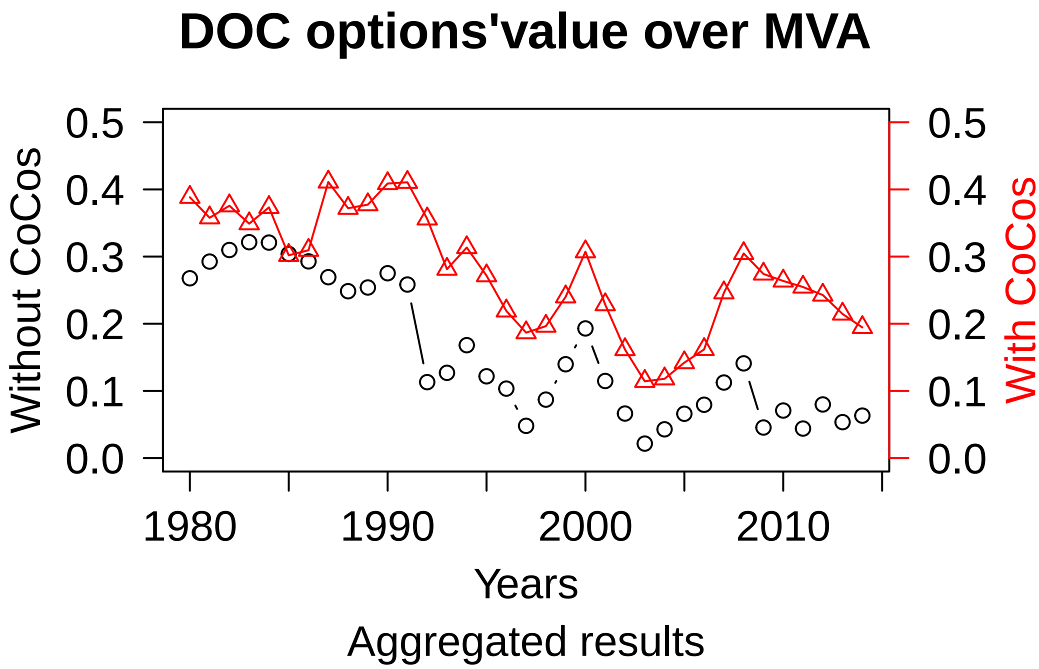


Figure 9: A comparison of the DOC option over MVA with and without CoCos.

In the context of a cluster analysis, cluster $2a$ is characterized by a higher franchise value, optimal volatility and leverage but lower policy rate relative to the cluster $2b$. We argue that in the cluster $2a$, enlarging the $Tier1$ through an average issue of CoCos of 5% in the US (or 3% in Europe), the DOC option is promoted over the default put option. We argue this is a first reason why in this cluster the average optimal volatility is relative higher (even if the median figure is aligned with the one in cluster $2b$) and the optimal policy rate is relative lower. The optimal policy rate figure does not consider the inflation and other macro issues.

10.3 Empirical results cluster by cluster

In the following paragraphs we assess what happens when we further apply cluster analysis to our data by leverage obtaining the following clusters: $2a1$, $2a2$, $2a3$ and $2b1$, $2b2$, $2b3$. Cluster $2a1$ (or $2b1$) have the highest capitalization, thus the largest $Tier1$ and represent our over capitalized clusters for which is optimal to issue (or not) CoCos, instead cluster $2a3$ (or $2b3$) have the lowest one, being our undercapitalized clusters. The clustering is defined on the basis of actual leverage in Basel terms, considering from one side the $Tier1$ figure reported by the banks, on the other side the total exposure given by the market value of the assets and franchise value obtained in the first step of the optimization procedure. Hence we show in the following table the value for leverage of our clusters.

Table 8: Leverage figures, cluster analysis.

Clusters	1	2	3
2a	0.1257	0.0850	0.0507
2b	0.1231	0.0872	0.0613
Aggregate	0.1112	0.0741	0.0429

This table shows us a relevant issue for the regulator. Taking into account the franchise value, at the aggregate level the under capitalized cluster is truly under capitalized for US regulation because leverage is below 5%,we show it to be almost 4.3%. This is corroborating our model results for the cluster $2a$ for which it is optimal to issue CoCos. In the next figure we show the optimal amount of CoCos to issue cluster by cluster. After 2000, this figure moves in the same direction for all the three clusters a part from the last year considered where the estimates diverge a lot where the average capitalized cluster demands the highest quantity of CoCos. Over the whole time span considered, the largest swings are present in $cluster2a3$, which is the under capitalized cluster. In particular, we show that right before the last financial crisis, it would have been optimal to issue CoCos for those under capitalized banks. It is only in the last year considered that they increase the $Tier1$ and we derive in our model an embedded market value of the assets and franchise value lower with respect to previous times[38].

[38]More on this in figure 13 presented in the appendix.

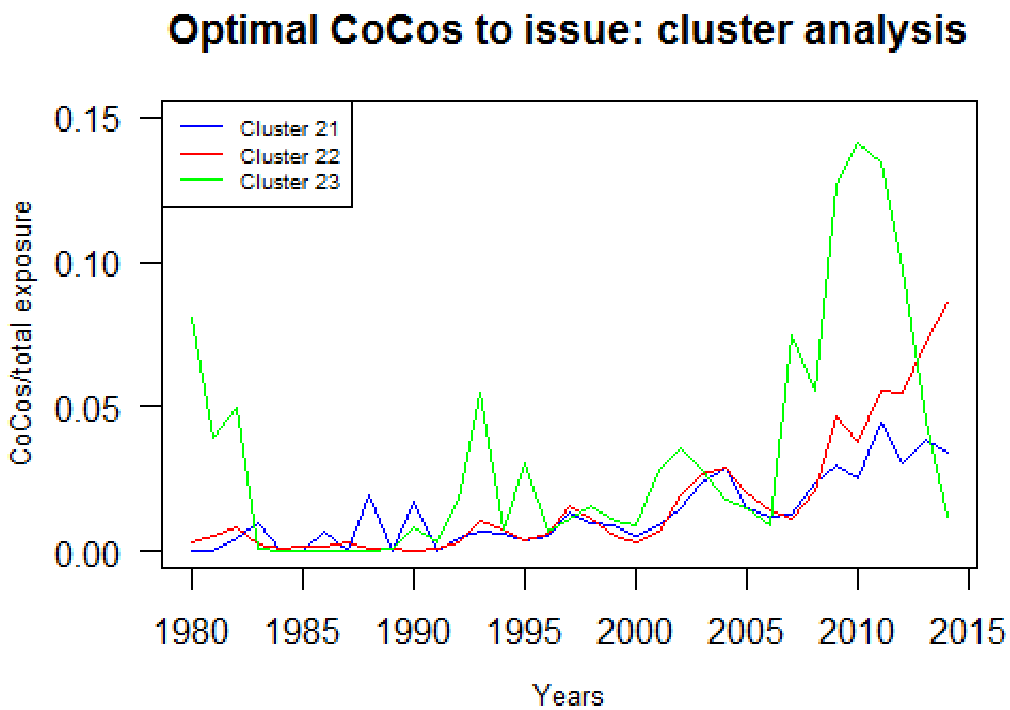


Figure 10: Optimal amount of CoCos over the market value of the assets and franchise value to issue per year, cluster by cluster.

Overall we notice that in the clusters for which is optimal to issue CoCos, going from the most capitalized cluster to the least one, the amount of franchise value increases, the optimal volatility decreases and the policy rate decreases as well. In example, the most under capitalized cluster would favour the default put option more with respect to the other two, because of the nature itself of the cluster. Thus, in order to maintain the franchise value and to promote it, given that the DOC is part of the optimization procedure, the other decision variables optimally adapt to preserve the overall optimized value of the whole objective function. Another relevant result is that the average optimal amount of leverage do not change on average, but differences emerges considering median optimal results, even if this variation is not so important with respect to the change in the other variables. Hence, we underline through these results that even if the CoCos contribution is important for the objective function maximization, the differences in the risk appetite among the clusters are mainly driven by the other optimizing variables. The clusters for which is not optimal to issue CoCos report always a lower franchise value, because the other variables (volatility and policy rate) are not enough to promote the DOC option as in the case for which the leverage is increased by the CoCos' issue. In the figure below we represent the optimized objective function per year, cluster by cluster, comparing the banks for which is optimal to issue CoCos and the ones for which is not optimal. The greater difference among the two emerges in the first cluster, this is due to the higher relative value in the optimized key variables. In third cluster, which is the most under capitalized one, after the financial crisis of 2008 the optimized objective functions diverge a lot between the two categories (with and without CoCos). This results is driven by the DOC option as we can see in Figure 11.

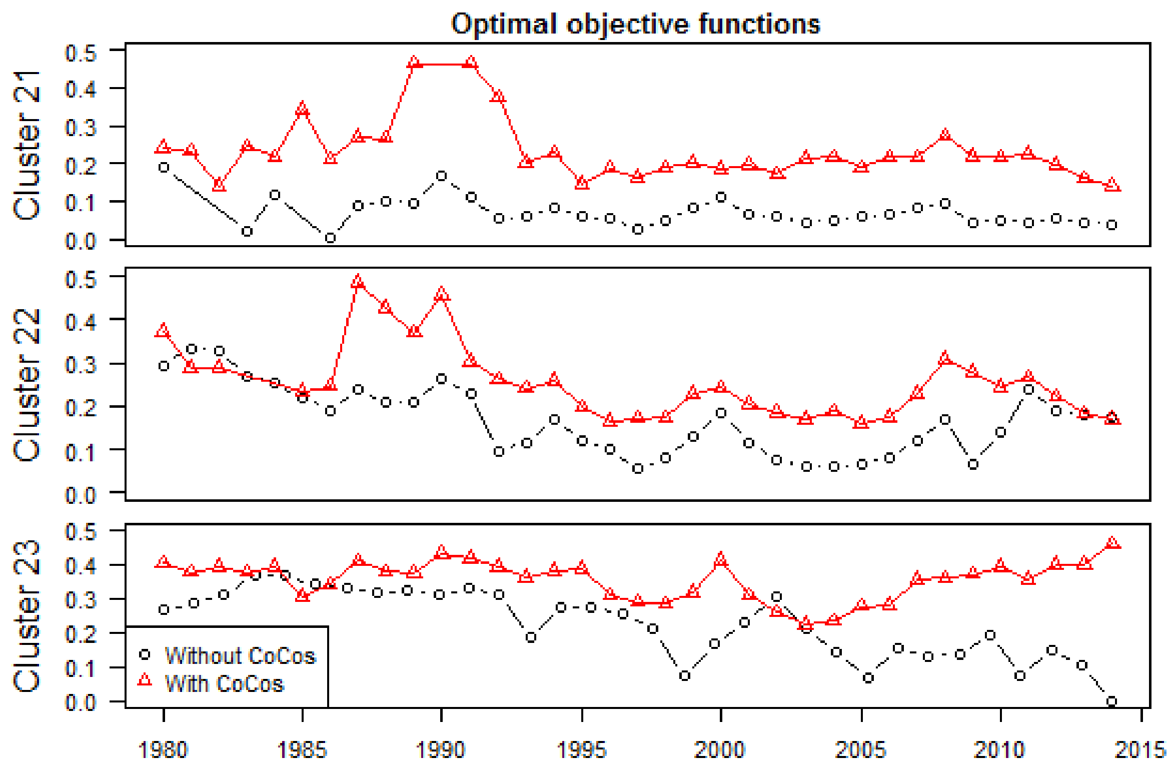


Figure 11: Optimal objective function: a comparison between the banks for which is optimal to issue CoCos and the ones for which is not optimal, per year, cluster by cluster.

In Figure 12, we display the DOC option, which represents the net present value of the growth opportunities for the banks we consider. The results are driven to zero only during the oil crisis in the '80s. Nevertheless, the greatest difference between the two optimal financing strategies drives upwards the DOC especially in the third cluster. Given the information obtained through the results above we do not attribute the whole explanation to the difference in the financing but also to the differences in the optimized values of the other decision variables.

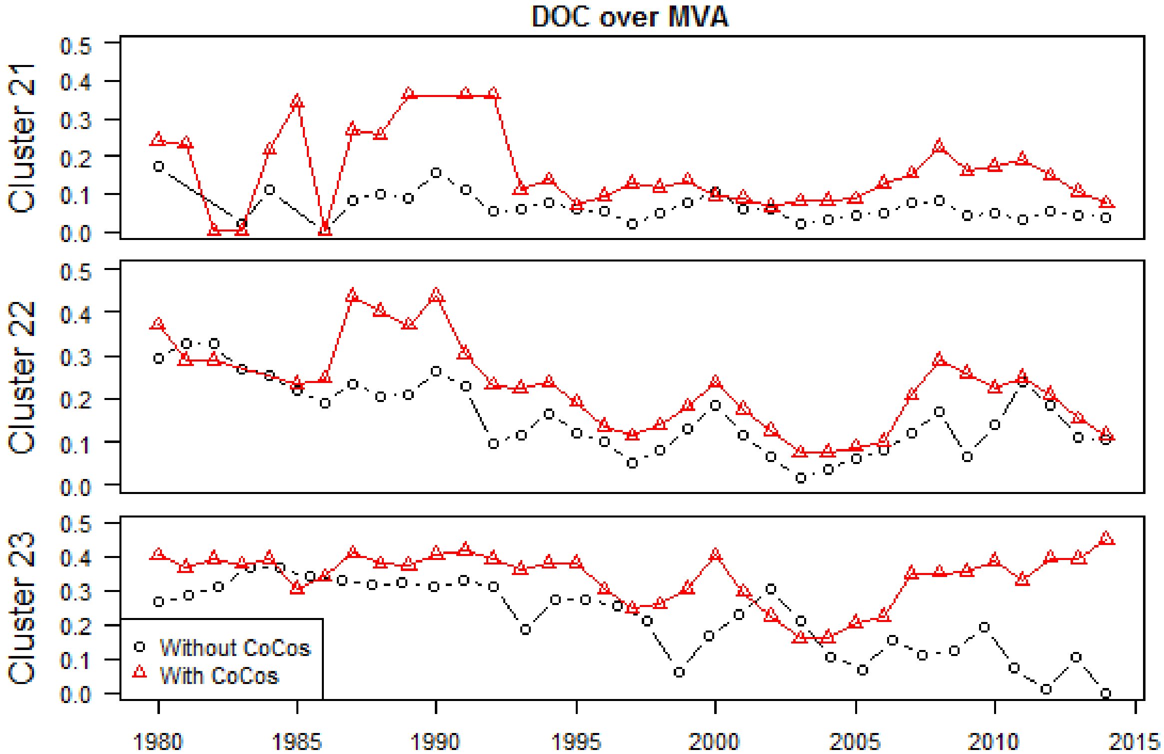


Figure 12: DOC: a comparison between the banks for which is optimal to issue CoCos and the ones for which is not optimal, per year, cluster by cluster.

10.4 The world after conversion with different policy rates

The results presented above are before a potential trigger and conversion event. What happens if the value of the underlying falls into the trigger area? First of all, the optimal amount of CoCos to issue is on average 3% for all the clusters contemplated, which is relatively small compared to the total amount of leverage, thus they contribute to the overall objective function maximization but with a bounded power. Second, the above results are valid for a pre-conversion scenario or if after conversion the value of the converted CoCos is the same. This means that if the delta conversion ratio is smaller than one, virtually, the pre-conversion total amount of optimal CoCos to issue is above the value estimated above, consequently we can use our results. Third, considering the optimal amount of CoCos estimated above pre-conversion, we show below the impact of different conversion ratio (delta) on the optimized default put option, DOC option (both of them standardized for the market value of the assets and franchise value, which are inputs in the second optimization) and the objective function. In absolute terms, we notice that the optimized functions do not change much but the differences are substantial in relative terms. The differences in the behaviour are even more important when we consider different policy rates. We show the results in the following figures, considering differences in the policy rates. In Figure 13, we show the behaviour after conversion with an average optimal policy rate.

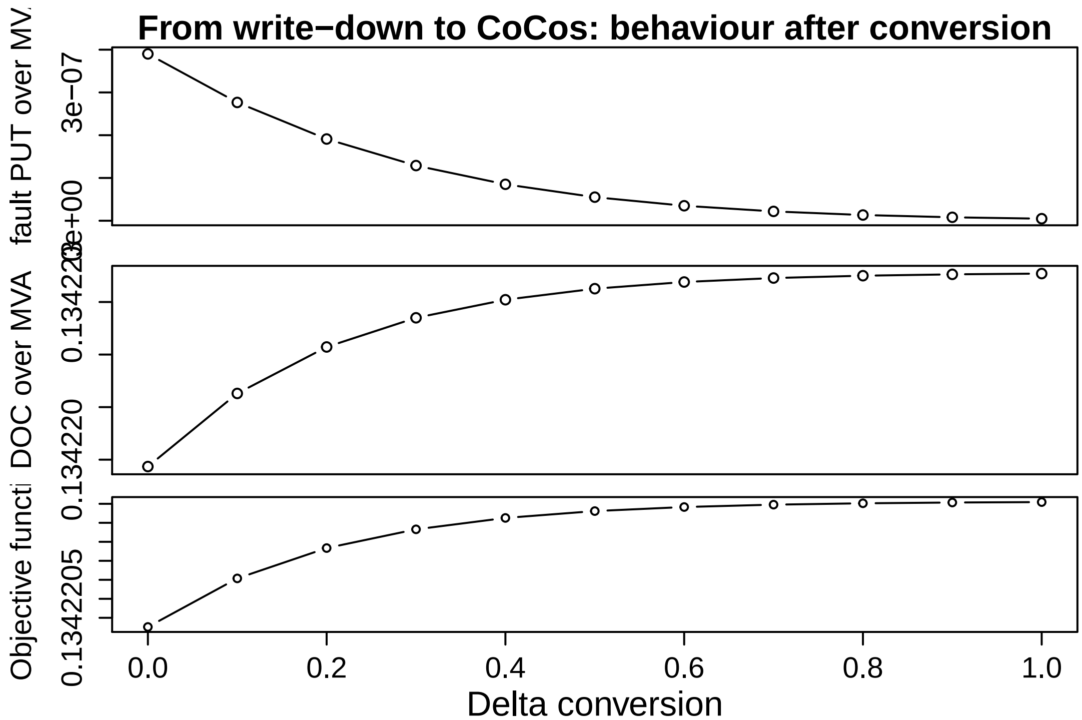


Figure 13: From write-down bonds to CoCos: behaviour after conversion, with an average optimal policy rate.

We assess the dynamics of our objective function in the zero environment for the policy rate. First, we exhibit in Figure 14 the dynamics with a generic policy rate equal to 0.5%. In this case, the default put option sensitivity presents a delayed decreasing behaviour. For the DOC option and the objective function, the directions are the same but there is more variability with respect to the one with average policy rate. Indeed, the objective function reaches its maximum after the case in which we use an average policy rate.

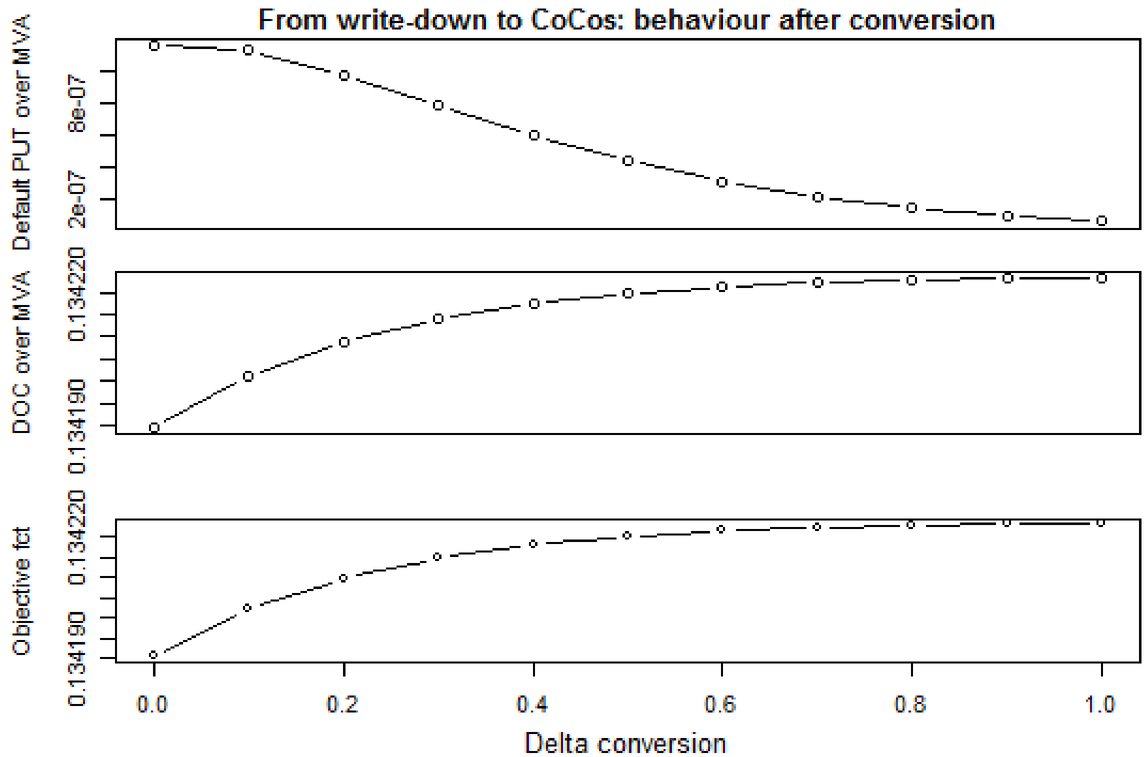


Figure 14: From write-down bonds to CoCos: behaviour after conversion, with a policy rate of 0.5%.

Second, we introduce the negative side of the policy rates, performing a sensitivity analysis with a rate of −0.5%, as reported in Figure 15.

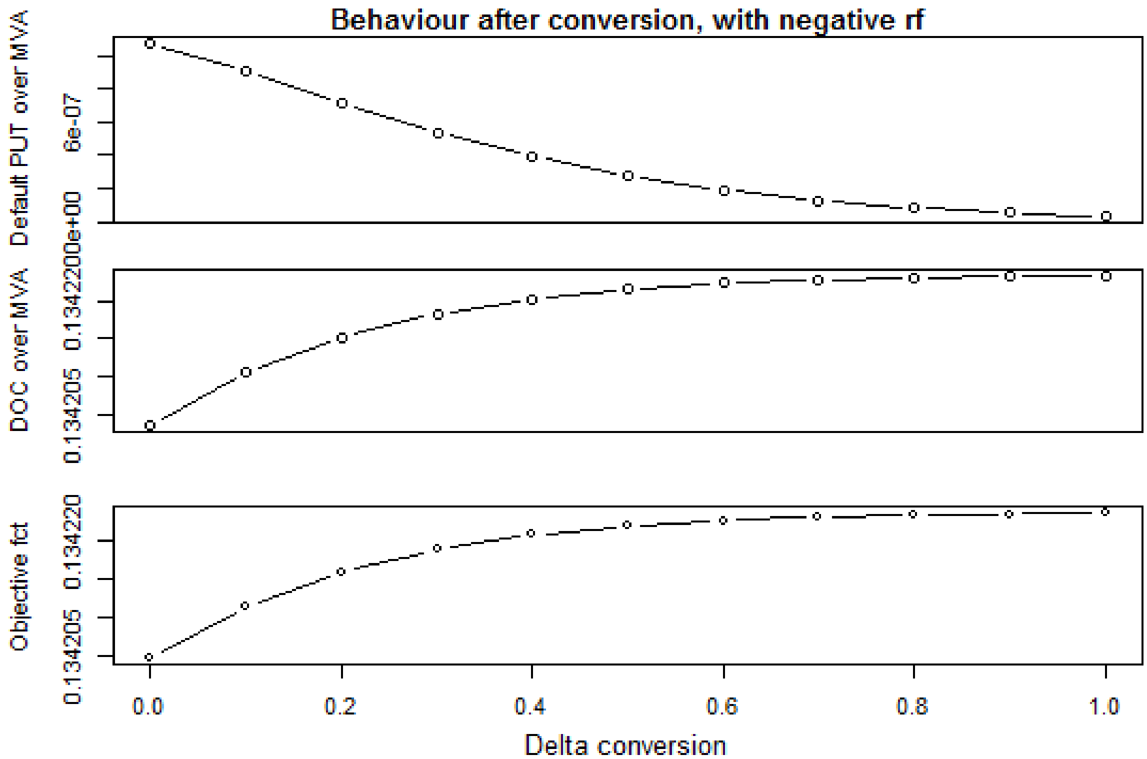


Figure 15: From write-down bonds to CoCos: behaviour after conversion, with a policy rate of -0.5%.

In this last case we can see that the dynamics for the default put option is again decreasing from the beginning, but the speed with which the objective function reaches the maximum is slowed down. Hence, in this section, we learn that the conversion ratio has an impact on the shape of the elements that accrue to the objective function. Nevertheless, the greatest determinant of the shape of the objective function remains the DOC option, dominating over the default put one.

11 Extensions

In our model, we consider an increase in the leverage, *ceteris paribus*, given by an issue of CoCos. We do not contemplate the opportunity of issuing equity because of the following concerns. First, hybrid capital, such as CoCo bonds, are treated as debt before conversion from a fiscal point of view, incentivizing CoCos issue through the benefits from tax shield. This is granted in Europe, it is instead more difficult in the US, given the strict parameters under which they are eligible for being considered debt before conversion for the US federal income tax purposes Hammer et al.

(2011). Second, following the literature, Myers (1984) and Myers and Majluf (1984) propose the pecking order theory, where firms issue equity as a "last resort", hence, it is not an ideal candidate for recapitalization. Stein (1992) refer to convertible bonds as a "backdoor equity financing". He emphasizes the feature of the call provision and reports that firms where the dividends are less than after-tax interest payments, conversion takes place earlier, with respect to the opposite case. This is interesting to motivate our following example. In our model, we consider the possibility to pay a discretionary CoCos' coupon in case conversion does not take place at time t: $c^{CoCo}1_{\{t<\tau_{Conv}\}}$. Shifting our focus from the balance sheet perspective to the income statement and to the cash flow (CF), we consider as a cost before taxes our coupon and thus, at least conforming to the European fiscal regulation, the bank enjoys the tax shield over the tax deductible coupon. We compare the following two income statement for a bank producing the same revenue, facing the same operational expenses structure and differing only for the financing[39]. In the following table we consider the case in which both coupons and dividends are payout.

Table 9: Bank income statements: a comparison between equity financing and CoCos financing.

Variables	Income statement and CF - bank with equity financing	Income statement and CF - bank with CoCos financing
Revenue	R	R
Operational Expenses	E	E
EBIT	R-E	R-E
Interest payments	0	c^{CoCo}
EBT	R-E	R-E-c^{CoCo}
Tax	$\eta(R-E)=\psi$	$\eta(R-E-c^{CoCo})=\psi-\eta(c^{CoCo})$
Net Income	$R-E-\psi$	$R-E-\psi+\eta(c^{CoCo})$
Dividens	d_{Eq}	d_{CoCo}
Cash position[40]	$IC-R-E-\psi-d_{Eq}$	$IC-R-E-\psi+\eta(c^{CoCo})-d_{CoCo}$

Tax shield and coupon savings is greater for CoCos even if compared to subordinated debt, given the smaller coupon rate. The difficulties in the tax deductibility in the US is one of the main reasons why in our sample, none of the banks issued CoCos in the time span considered. On the other

[39]Please note that the variable R is positive and greater than the whole amount of expenses E. We are considering a bank whose net income is very low but still greater than zero. Interest payments for the bank opting for recapitalization via equity are zero because straight debt is considered to be a zero-coupon, as explained in the main assumptions of this model.

side, the fiscal regulation in Europe allowing for tax shield facilitated a massive issue of CoCos in response to the financial crisis. In example, we can see empirically an increasing market for CoCos after Lloyds Banking Group 2009 first issue. We show that the cash position even if impaired by the CoCos' coupon payout enjoys the savings due to the tax shield and the lower dividend to be payout[41] Third, CoCos' coupons are discretionary, if the CoCos are eligible as $AT1$. In case of subsequent bank value deterioration, the bank might choose not to pay the coupon, which is on average 7%, which is always greater than the average dividend yield, which is on average for the banking industry 1.63%[42]. In case of bad times the bank might decide to save the coupon amount, since it is reasonable that during bad times dividends are zero or very low and it would be incoherent to payout high CoCos coupons. This choice accrues to the market value of the bank and returns a greater saving amount with respect to the saving of the dividend global amount to payout. In the next table we compare the same two banks proposed in the example above, with neither the CoCos' coupon payment nor the dividend payment and we propose a summary of the potential savings.

Table 10: Bank savings and Cash improvement: a comparison between equity financing and CoCos financing.

Variables	Income statement and CF - bank with equity financing	Income statement and CF - bank with CoCos financing
Revenue	R	R
Operational Expenses	E	E
EBIT	R-E	R-E
Interest payments	0	0
EBT	R-E	R-E
Tax	$\eta(R-E)=\psi$	$\eta(R-E-0)=\psi$
Net Income	$R-E-\psi$	$R-E-\psi$
Dividends	0	0
Savings/Cash improvement[43]	d_{Eq}	$d_{CoCo}+c^{CoCo}$
Cash position with savings[44]	$IC-R-E-\psi+d_{Eq}$	$IC-R-E-\psi+d_{CoCo}+c^{CoCo}$

In this case the two banks looks identical from a profit and loss point of view, but the cash flows of the two differ substantially. In the first case the cash flow is improved by an amount of d_{Eq}, which is the amount of dividends the bank opting for standard equity recapitalization is able to save

[41] Even if the dividend for the bank financed via equity is lower with respect to the CoCos' coupon.

[42] Data are obtained from the (2014) and the A. Damodaran website: $http://people.stern.nyu.edu/adamodar/New_Home_Page/home.htm$.

deciding not to payout dividends. The bank, enlarging its $Tier1$, through a CoCos issue, if opt for not paying the discretionary coupon would have paid to its shareholders a dividend equal to the one of the other bank. Up to this point the amount of savings and consequent improvement of the cash position is the same for the two banks. Nevertheless, the bank, that has issued CoCos, enjoys a further saving equal to the coupon amount that would have been payout. We do not consider bankruptcy costs because we assess the bank risk appetite behaviour when assets are deteriorating but the bank is still able to run its assets and we concentrate on understanding which is the best set of instruments in order to preserve the franchise value. We do not compare the subordinated debt in this case because it does not accrue to the $Tier1$. Subordinated debt posticipates default, but does not play a role over the franchise barrier. Subordinated debt is a $Tier2$ instruments and frees resources to bank only for avoiding default, thus after that our franchise value would have been eroded. Hence, the default barrier level is the same if a bank opt for a CoCo or subordinated debt issue. Overall, the default put option value, *ceteris paribus*, do not change, but, a subordinated deb issue would harm the DOC option, because the trigger barrier for franchise value would be relative higher with respect to a CoCos' issue.

12 Conclusion

The financial crisis of 2007-2008 has been the greatest global financial crisis since the Great Depression. Therefore, regulators and governments all over the world favoured the issuance of contingent capital instruments, being potentially a useful tool for strengthening banks' capital positions and for facing losses, as highlighted in the Financial Stability Oversight Council Report Council (2012). In this context, we investigate the shape of risk appetite when the bank is financed also with contingent convertible bonds (CoCos). In our model, the manager acts in order to maximize the bank objective function, given by the sum of the default put option and the down-and-out call option, pricing the net present value of growth opportunities. The optimization procedure adjusts jointly the assets and franchise value volatility, the policy rate, the level of leverage, and the consequent level of CoCos to issue. Risk appetite is given by the first order derivatives. We find that both our model and Basel III recommendations converge over their incentives regarding volatility and leverage. We show that for banks with higher franchise value it is optimal to issue an average amount of CoCos over the market value of the assets of 3%. CoCos do not only enlarge the distance to the default barrier but also the franchise value one. The optimized objective function with CoCos is always higher than the one without CoCos, this is true both at aggregate level and in the cluster analysis. On the optimal volatility side, a decrease accrue to the DOC option, in particular for under capitalized banks, even if this figure can't be pushed too low in order to preserve the default put option. This can be obtained by changing assets' composition and growth opportunities strategies. The other optimizing decision variables are crucial in shaping risk appetite and help in understanding the differences among the clusters. We do not differentiate ex-ante with respect to conversion between write down bonds and CoCos but only afterwards. After conversion for the bank is always optimal to have a conversion ratio greater than zero even if loss absorbing, thus being smaller than one. In absolute terms, we notice that the optimized functions do not change much in value, but the differences are substantial in relative terms. The differences in the behaviour are even more important when we consider different policy rates. Our contribution to the existent literature is to assess risk appetite in a multi-dimensional perspective and to account for differences among banks' clusters. We show that those peculiarities are even more important when accounting for CoCos and policy rates approaching zero or negative figures, in a Basel III friendly framework. A flat regulation could harm certain categories of banks, especially considering volatility. On the policy rate side, we argue that at aggregate level the average optimal rate for stability is far from the optimal figure at cluster level. Both at aggregate and cluster level the resulting optimal policy rate is larger with respect to the actual figure. This is particularly true in the last period of the time span considered. This element might be taken into consideration in relative terms, since we do not account for other key figures addressed by monetary policy.

Appendix

First order, second order and joint derivatives of the objective function

In this paragraph, for exposition reasons, leverage is defined as the ratio between the market value of straight debt and the market value of the assets together with the franchise value. This choice is for exposition reasons.

$$\text{ObjectiveFunction}\left(lev, sig\_A\_Fr, rf\right) :=$$

$$lev\ (MVA + Fr)\ \text{cdf\_normal}\left(\frac{-\left(\log\left(\frac{1}{lev}\right) + \left(rf - 0.5\, sig\_A\_Fr^2\right)\, T\right)}{sig\_A\_Fr\,\sqrt{T}}, 0, 1\right) +$$
$$-\ (MVA + Fr)\ \text{cdf\_normal}\left(\frac{-\left(\log\left(\frac{1}{lev}\right) + \left(rf + 0.5\, sig\_A\_Fr^2\right)\, T\right)}{sig\_A\_Fr\,\sqrt{T}}, 0, 1\right) +$$
$$+\ Fr\ \left(\text{cdf\_normal}\left(\frac{\log\left(\frac{1}{lev}\right)}{sig\_A\_Fr\,\sqrt{T}} + sig\_A\_Fr\,\sqrt{T}\,\frac{rf + 0.5\, sig\_A\_Fr^2}{sig\_A\_Fr^2}, 0, 1\right)\right) +$$
$$-\ Fr\ \left(lev^{2\,\frac{rf+0.5\, sig\_A\_Fr^2}{sig\_A\_Fr^2}}\ \text{cdf\_normal}\left(\frac{\log(lev)}{sig\_A\_Fr\,\sqrt{T}} + sig\_A\_Fr\,\sqrt{T}\,\frac{rf + 0.5\, sig\_A\_Fr^2}{sig\_A\_Fr^2}, 0, 1\right)\right)$$

First order derivative with respect to leverage:

$$(MVA+Fr)\left(\frac{\operatorname{erf}\left(\frac{\log(lev)-\left(rf-0.5\,sig\_A\_Fr^2\right)T}{\sqrt{2}\,sig\_A\_Fr\,\sqrt{T}}\right)}{2}+\frac{1}{2}\right)-\frac{(MVA+Fr)\,e^{-\frac{\left(\log(lev)-\left(0.5\,sig\_A\_Fr^2+rf\right)T\right)^2}{2\,sig\_A\_Fr^2\,T}}}{\sqrt{2}\,\sqrt{\pi}\,lev\,sig\_A\_Fr\,\sqrt{T}}+$$

$$+\frac{(MVA+Fr)\,e^{-\frac{\left(\log(lev)-\left(rf-0.5\,sig\_A\_Fr^2\right)T\right)^2}{2\,sig\_A\_Fr^2\,T}}}{\sqrt{2}\,\sqrt{\pi}\,sig\_A\_Fr\,\sqrt{T}}+Fr$$

$$(-\frac{lev^{\frac{2\left(0.5\,sig\_A\_Fr^2+rf\right)}{sig\_A\_Fr^2}-1}\,e^{-\frac{\left(\frac{\left(0.5\,sig\_A\_Fr^2+rf\right)\sqrt{T}}{sig\_A\_Fr}+\frac{\log(lev)}{sig\_A\_Fr\,\sqrt{T}}\right)^2}{2}}}{\sqrt{2}\,\sqrt{\pi}\,sig\_A\_Fr\,\sqrt{T}}+$$

$$-\frac{e^{-\frac{\left(\frac{\left(0.5\,sig\_A\_Fr^2+rf\right)\sqrt{T}}{sig\_A\_Fr}-\frac{\log(lev)}{sig\_A\_Fr\,\sqrt{T}}\right)^2}{2}}}{\sqrt{2}\,\sqrt{\pi}\,lev\,sig\_A\_Fr\,\sqrt{T}}-\frac{2\,lev^{\frac{2\left(0.5\,sig\_A\_Fr^2+rf\right)}{sig\_A\_Fr^2}-1}\left(0.5\,sig\_A\_Fr^2+rf\right)\left(\frac{\operatorname{erf}\left(\frac{\frac{\left(0.5\,sig\_A\_Fr^2+rf\right)\sqrt{T}}{sig\_A\_Fr}+\frac{\log(lev)}{sig\_A\_Fr\,\sqrt{T}}}{\sqrt{2}}\right)}{2}+\frac{1}{2}\right)}{sig\_A\_Fr^2}$$

Vega, first order derivative with respect to volatility:

$$-\frac{(MVA+Fr)\left(-\frac{\log(lev)-\left(0.5\,sig\_A\_Fr^2+rf\right)T}{\sqrt{2}\,sig\_A\_Fr^2\,\sqrt{T}}-\frac{1.0\sqrt{T}}{\sqrt{2}}\right)e^{-\frac{\left(\log(lev)-\left(0.5\,sig\_A\_Fr^2+rf\right)T\right)^2}{2\,sig\_A\_Fr^2\,T}}}{\sqrt{\pi}}+$$

$$+\frac{lev\,(MVA+Fr)\left(\frac{1.0\sqrt{T}}{\sqrt{2}}-\frac{\log(lev)-\left(rf-0.5\,sig\_A\_Fr^2\right)T}{\sqrt{2}\,sig\_A\_Fr^2\,\sqrt{T}}\right)e^{-\frac{\left(\log(lev)-\left(rf-0.5\,sig\_A\_Fr^2\right)T\right)^2}{2\,sig\_A\_Fr^2\,T}}}{\sqrt{\pi}}+$$

$$+Fr\,(-\frac{lev^{\frac{2\left(0.5\,sig\_A\_Fr^2+rf\right)}{sig\_A\_Fr^2}}\left(-\frac{\left(0.5\,sig\_A\_Fr^2+rf\right)\sqrt{T}}{sig\_A\_Fr^2}+1.0\sqrt{T}-\frac{\log(lev)}{sig\_A\_Fr^2\,\sqrt{T}}\right)e^{-\frac{\left(\frac{\left(0.5\,sig\_A\_Fr^2+rf\right)\sqrt{T}}{sig\_A\_Fr}+\frac{\log(lev)}{sig\_A\_Fr\,\sqrt{T}}\right)^2}{2}}}{\sqrt{2}\,\sqrt{\pi}}+$$

$$\frac{\left(-\frac{\left(0.5\,sig\_A\_Fr^2+rf\right)\sqrt{T}}{sig\_A\_Fr^2}+1.0\sqrt{T}+\frac{\log(lev)}{sig\_A\_Fr^2\,\sqrt{T}}\right)e^{-\frac{\left(\frac{\left(0.5\,sig\_A\_Fr^2+rf\right)\sqrt{T}}{sig\_A\_Fr}-\frac{\log(lev)}{sig\_A\_Fr\,\sqrt{T}}\right)^2}{2}}}{\sqrt{2}\,\sqrt{\pi}}+$$

$$-\,lev^{\frac{2\left(0.5\,sig\_A\_Fr^2+rf\right)}{sig\_A\_Fr^2}}\log(lev)\left(\frac{2.0}{sig\_A\_Fr}-\frac{4\left(0.5\,sig\_A\_Fr^2+rf\right)}{sig\_A\_Fr^3}\right)\left(\frac{\operatorname{erf}\left(\frac{\frac{\left(0.5\,sig\_A\_Fr^2+rf\right)\sqrt{T}}{sig\_A\_Fr}+\frac{\log(lev)}{sig\_A\_Fr\,\sqrt{T}}}{\sqrt{2}}\right)}{2}+\frac{1}{2}\right))$$

Rho, first order derivative with respect to the policy rate:

$$\frac{(MVA+Fr)\,\sqrt{T}\,e^{-\frac{\left(\log(lev)-\left(0.5\,sig\_A\_Fr^{2}+rf\right)T\right)^{2}}{2\,sig\_A\_Fr^{2}\,T}}}{\sqrt{2}\,\sqrt{\pi}\,sig\_A\_Fr}-\frac{lev\,(MVA+Fr)\,\sqrt{T}\,e^{-\frac{\left(\log(lev)-\left(rf-0.5\,sig\_A\_Fr^{2}\right)T\right)^{2}}{2\,sig\_A\_Fr^{2}\,T}}}{\sqrt{2}\,\sqrt{\pi}\,sig\_A\_Fr}+$$

$$+Fr\,(-\frac{lev^{\frac{2\left(0.5\,sig\_A\_Fr^{2}+rf\right)}{sig\_A\_Fr^{2}}}\,\sqrt{T}\,e^{-\frac{\left(\frac{\left(0.5\,sig\_A\_Fr^{2}+rf\right)\sqrt{T}}{sig\_A\_Fr}+\frac{\log(lev)}{sig\_A\_Fr\,\sqrt{T}}\right)^{2}}{2}}}{\sqrt{2}\,\sqrt{\pi}\,sig\_A\_Fr}+\frac{\sqrt{T}\,e^{-\frac{\left(\frac{\left(0.5\,sig\_A\_Fr^{2}+rf\right)\sqrt{T}}{sig\_A\_Fr}-\frac{\log(lev)}{sig\_A\_Fr\,\sqrt{T}}\right)^{2}}{2}}}{\sqrt{2}\,\sqrt{\pi}\,sig\_A\_Fr}-$$

$$\frac{2\,lev^{\frac{2\left(0.5\,sig\_A\_Fr^{2}+rf\right)}{sig\_A\_Fr^{2}}}\,\log(lev)\left(\frac{\operatorname{erf}\left(\frac{\frac{\left(0.5\,sig\_A\_Fr^{2}+rf\right)\sqrt{T}}{sig\_A\_Fr}+\frac{\log(lev)}{sig\_A\_Fr\,\sqrt{T}}}{\sqrt{2}}\right)}{2}+\frac{1}{2}\right)}{sig\_A\_Fr^{2}})$$

First joint derivative with respect to both leverage and volatility:

$$\frac{d^{sig\_A\_Fr}}{d\,lev^{sig\_A\_Fr}}\left(-(MVA+Fr)\left(\frac{\operatorname{erf}\left(\frac{\log(lev)-\left(0.5\,sig\_A\_Fr^{2}+rf\right)T}{\sqrt{2}\,sig\_A\_Fr\,\sqrt{T}}\right)}{2}+\frac{1}{2}\right)+lev\,(MVA+Fr)\left(\frac{\operatorname{erf}\left(\frac{\log(lev)-\left(rf-0.5\,sig\_A\_Fr^{2}\right)T}{\sqrt{2}\,sig\_A\_Fr\,\sqrt{T}}\right)}{2}+\frac{1}{2}\right)\right)+$$

$$+\frac{d^{sig\_A\_Fr}}{d\,lev^{sig\_A\_Fr}}\left(Fr\left(-lev^{\frac{2\left(0.5\,sig\_A\_Fr^{2}+rf\right)}{sig\_A\_Fr^{2}}}\left(\frac{\operatorname{erf}\left(\frac{\frac{\left(0.5\,sig\_A\_Fr^{2}+rf\right)\sqrt{T}}{sig\_A\_Fr}+\frac{\log(lev)}{sig\_A\_Fr\,\sqrt{T}}}{\sqrt{2}}\right)}{2}+\frac{1}{2}\right)\right)\right)+$$

$$+\frac{d^{sig\_A\_Fr}}{d\,lev^{sig\_A\_Fr}}\left(Fr\left(+\frac{\operatorname{erf}\left(\frac{\frac{\left(0.5\,sig\_A\_Fr^{2}+rf\right)\sqrt{T}}{sig\_A\_Fr}-\frac{\log(lev)}{sig\_A\_Fr\,\sqrt{T}}}{\sqrt{2}}\right)}{2}+\frac{1}{2}\right)\right)$$

Second joint derivative with respect to both leverage and policy rate:

$$\frac{d^{rf}}{d\,lev^{rf}}\left(-(MVA+Fr)\left(\frac{\operatorname{erf}\left(\frac{\log(lev)-\left(0.5\,sig\_A\_Fr^{2}+rf\right)T}{\sqrt{2}\,sig\_A\_Fr\,\sqrt{T}}\right)}{2}+\frac{1}{2}\right)+lev\,(MVA+Fr)\left(\frac{\operatorname{erf}\left(\frac{\log(lev)-\left(rf-0.5\,sig\_A\_Fr^{2}\right)T}{\sqrt{2}\,sig\_A\_Fr\,\sqrt{T}}\right)}{2}+\frac{1}{2}\right)\right)+$$

$$+\frac{d^{rf}}{d\,lev^{rf}}\left(+Fr\left(-lev^{\frac{2\left(0.5\,sig\_A\_Fr^{2}+rf\right)}{sig\_A\_Fr^{2}}}\left(\frac{\operatorname{erf}\left(\frac{\frac{\left(0.5\,sig\_A\_Fr^{2}+rf\right)\sqrt{T}}{sig\_A\_Fr}+\frac{\log(lev)}{sig\_A\_Fr\,\sqrt{T}}}{\sqrt{2}}\right)}{2}+\frac{1}{2}\right)+\frac{\operatorname{erf}\left(\frac{\frac{\left(0.5\,sig\_A\_Fr^{2}+rf\right)\sqrt{T}}{sig\_A\_Fr}-\frac{\log(lev)}{sig\_A\_Fr\,\sqrt{T}}}{\sqrt{2}}\right)}{2}+\frac{1}{2}\right)\right)$$

Third joint derivative with respect to the policy rate and volatility:

$$\frac{d^{rf}}{d\, sig\_A\_Fr^{rf}}\left(-(MVA+Fr)\left(\frac{\operatorname{erf}\left(\frac{\log(lev)-\left(0.5\, sig\_A\_Fr^2+rf\right)T}{\sqrt{2}\, sig\_A\_Fr\,\sqrt{T}}\right)}{2}+\frac{1}{2}\right)+lev\,(MVA+Fr)\left(\frac{\operatorname{erf}\left(\frac{\log(lev)-\left(rf-0.5\, sig\_A\_Fr^2\right)T}{\sqrt{2}\, sig\_A\_Fr\,\sqrt{T}}\right)}{2}+\frac{1}{2}\right)\right)+$$

$$+\frac{d^{rf}}{d\, sig\_A\_Fr^{rf}}\left(+Fr\left(-lev^{\frac{2\left(0.5\, sig\_A\_Fr^2+rf\right)}{sig\_A\_Fr^2}}\left(\frac{\operatorname{erf}\left(\frac{\frac{\left(0.5\, sig\_A\_Fr^2+rf\right)\sqrt{T}}{sig\_A\_Fr}+\frac{\log(lev)}{sig\_A\_Fr\,\sqrt{T}}}{\sqrt{2}}\right)}{2}+\frac{1}{2}\right)\right)\right)+$$

$$+\frac{d^{rf}}{d\, sig\_A\_Fr^{rf}}\left(+Fr\left(+\frac{\operatorname{erf}\left(\frac{\frac{\left(0.5\, sig\_A\_Fr^2+rf\right)\sqrt{T}}{sig\_A\_Fr}-\frac{\log(lev)}{sig\_A\_Fr\,\sqrt{T}}}{\sqrt{2}}\right)}{2}+\frac{1}{2}\right)\right)$$

Second-order derivative with respect to leverage:

$$\frac{d^{lev}}{d\, lev^{lev}}\left(-(MVA+Fr)\left(\frac{\operatorname{erf}\left(\frac{\log(lev)-\left(0.5\, sig\_A\_Fr^2+rf\right)T}{\sqrt{2}\, sig\_A\_Fr\,\sqrt{T}}\right)}{2}+\frac{1}{2}\right)+lev\,(MVA+Fr)\left(\frac{\operatorname{erf}\left(\frac{\log(lev)-\left(rf-0.5\, sig\_A\_Fr^2\right)T}{\sqrt{2}\, sig\_A\_Fr\,\sqrt{T}}\right)}{2}+\frac{1}{2}\right)\right)+$$

$$+\frac{d^{lev}}{d\, lev^{lev}}\left(+Fr\left(-lev^{\frac{2\left(0.5\, sig\_A\_Fr^2+rf\right)}{sig\_A\_Fr^2}}\left(\frac{\operatorname{erf}\left(\frac{\frac{\left(0.5\, sig\_A\_Fr^2+rf\right)\sqrt{T}}{sig\_A\_Fr}+\frac{\log(lev)}{sig\_A\_Fr\,\sqrt{T}}}{\sqrt{2}}\right)}{2}+\frac{1}{2}\right)+\frac{\operatorname{erf}\left(\frac{\frac{\left(0.5\, sig\_A\_Fr^2+rf\right)\sqrt{T}}{sig\_A\_Fr}-\frac{\log(lev)}{sig\_A\_Fr\,\sqrt{T}}}{\sqrt{2}}\right)}{2}+\frac{1}{2}\right)\right)$$

Vomma, second-order derivative with respect to volatility:

$$\frac{d^{sig\_A\_Fr}}{d\, sig\_A\_Fr^{sig\_A\_Fr}}\left(-(MVA+Fr)\left(\frac{\operatorname{erf}\left(\frac{\log(lev)-\left(0.5\, sig\_A\_Fr^2+rf\right)T}{\sqrt{2}\, sig\_A\_Fr\,\sqrt{T}}\right)}{2}+\frac{1}{2}\right)+lev\,(MVA+Fr)\left(\frac{\operatorname{erf}\left(\frac{\log(lev)-\left(rf-0.5\, sig\_A\_Fr^2\right)T}{\sqrt{2}\, sig\_A\_Fr\,\sqrt{T}}\right)}{2}+\frac{1}{2}\right)\right)$$

$$\frac{d^{sig\_A\_Fr}}{d\, sig\_A\_Fr^{sig\_A\_Fr}}\left(+Fr\left(-lev^{\frac{2\left(0.5\, sig\_A\_Fr^2+rf\right)}{sig\_A\_Fr^2}}\left(\frac{\operatorname{erf}\left(\frac{\frac{\left(0.5\, sig\_A\_Fr^2+rf\right)\sqrt{T}}{sig\_A\_Fr}+\frac{\log(lev)}{sig\_A\_Fr\,\sqrt{T}}}{\sqrt{2}}\right)}{2}+\frac{1}{2}\right)\right)\right)+$$

$$+\frac{d^{sig\_A\_Fr}}{d\, sig\_A\_Fr^{sig\_A\_Fr}}\left(+Fr\left(\frac{\operatorname{erf}\left(\frac{\frac{\left(0.5\, sig\_A\_Fr^2+rf\right)\sqrt{T}}{sig\_A\_Fr}-\frac{\log(lev)}{sig\_A\_Fr\,\sqrt{T}}}{\sqrt{2}}\right)}{2}+\frac{1}{2}\right)\right)$$

Second-order derivative with respect to the policy rate:

$$\frac{d^{rf}}{d\,rf^{rf}}\left(-\left(MVA+Fr\right)\left(\frac{\operatorname{erf}\left(\frac{\log(lev)-\left(0.5\,sig\_A\_Fr^2+rf\right)T}{\sqrt{2}\,sig\_A\_Fr\,\sqrt{T}}\right)}{2}+\frac{1}{2}\right)+lev\,\left(MVA+Fr\right)\left(\frac{\operatorname{erf}\left(\frac{\log(lev)-\left(rf-0.5\,sig\_A\_Fr^2\right)T}{\sqrt{2}\,sig\_A\_Fr\,\sqrt{T}}\right)}{2}+\frac{1}{2}\right)\right)+$$

$$+\frac{d^{rf}}{d\,rf^{rf}}\left(+Fr\left(-lev^{\frac{2\left(0.5\,sig\_A\_Fr^2+rf\right)}{sig\_A\_Fr^2}}\left(\frac{\operatorname{erf}\left(\frac{\frac{\left(0.5\,sig\_A\_Fr^2+rf\right)\sqrt{T}}{sig\_A\_Fr}+\frac{\log(lev)}{sig\_A\_Fr\,\sqrt{T}}}{\sqrt{2}}\right)}{2}+\frac{1}{2}\right)+\frac{\operatorname{erf}\left(\frac{\frac{\left(0.5\,sig\_A\_Fr^2+rf\right)\sqrt{T}}{sig\_A\_Fr}-\frac{\log(lev)}{sig\_A\_Fr\,\sqrt{T}}}{\sqrt{2}}\right)}{2}+\frac{1}{2}\right)\right)$$

Who drives the risk appetite? A simulation exercise

In the following table we perform a sensitivity analysis to change in the three optimization variables. We comment in Section 6 how the shape of risk appetite differs among the clusters. Optimal value ranges:

- for leverage:

$$\begin{aligned} &Cluster \quad 21: \quad 0.10 \leq lev^* \leq 0.11 \\ &Cluster \quad 22: \quad 0.07 \leq lev^* \leq 0.08 \\ &Cluster \quad 23: \quad 0.06 \leq lev^* \leq 0.07 \end{aligned} \tag{39}$$

  FED benchmarks are:0.08 for 6 Systemically important financial institution banks; 0.05 for their insured bank holding firms.

- for volatility:

$$\begin{aligned} &Cluster \quad 21: \quad 0.04 \leq \sigma^* \leq 0.06 \\ &Cluster \quad 22: \quad 0.02 \leq \sigma^* \leq 0.04 \\ &Cluster \quad 23: \quad 0.01 \leq \sigma^* \leq 0.02 \end{aligned} \tag{40}$$

  Under-capitalized banks should have a relative lower volatility: higher probability to cross the barrier and the DOC to expire.

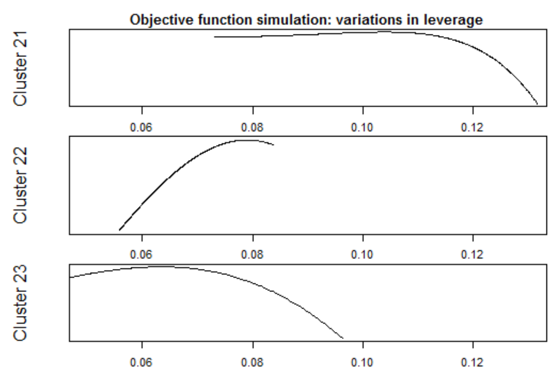
- for policy rate:

$$\begin{aligned}
Cluster \quad 21: &\quad 0.13 \leq rf^* \leq 0.15 \\
Cluster \quad 22: &\quad 0.11 \leq rf^* \leq 0.14 \\
Cluster \quad 23: &\quad 0.04 \leq rf^* \leq 0.07
\end{aligned} \tag{41}$$

We find relative higher optimal values because we do not consider economic growth and the simulation is based on input values derived from our empirical sample, time span (1980-2014).

A simulation exercise: which is the main driver of the objective function between the two options for leverage?

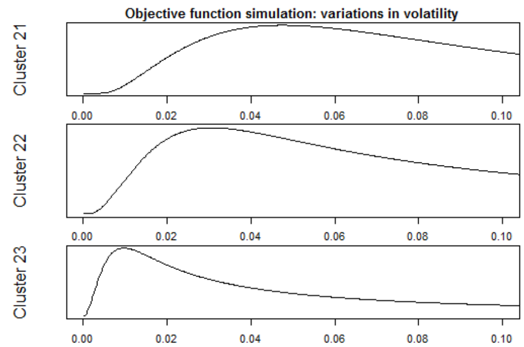
- The optimal value for leverage in all the three clusters is above the actual levels and slightly above FED recommendation.
- Our results take into account the franchise value (in the denominator of "leverage") and this is a main difference between our results and the regulator ones.
- Given the pricing of the DOC option and the definition of leverage, the DOC is maximized for lower levels of leverage.



A simulation exercise: which is the main driver of the objective function between the two options for the volatility?

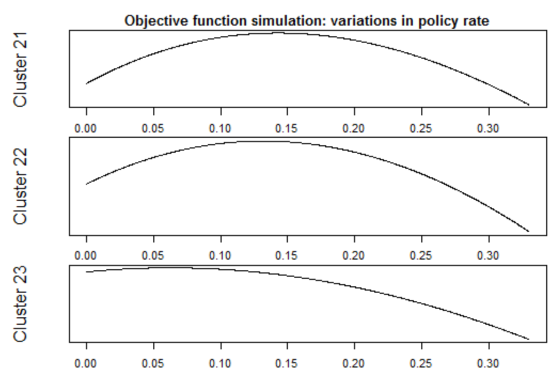
- The DOC option, being a barrier option, is optimized in our context for relative lower volatility values: when volatility is too high, there might be a breaching of the barrier and, consequently, a collapse of the franchise value.
- With smaller values of tier1, going from cluster 21 to 23, the peak of the optimized objective functions moves to the left.

- For relative smaller tier1, our barrier is much higher and smaller values of volatility ensure the franchise value preservation.



Which is the main driver of the objective function between the two options for the policy rate?

- The optimized objective function displays a concave shape, with minor differences in the peak depending on the cluster.
- Once it is clear our risk appetite specification, the policy maker change in the rate has a clear impact.
- Overall, our optimal policy rate results slightly high because in our model we do not consider economic growth and inflation.



Summary statistics and Confidence intervals for optimal value of the decision variables

We display in the following tables the summary statistics with values for actual amount of leverage that is adapted in order to be compared to our definition of optimal leverage which consider the franchise value, the market value of the assets and the market value of straight debt and deposits, but not the actual $Tier1$. The following table records the summary statistics for the key figures of the sub sample of banks for which is not optimal to issue CoCos.

Table 11: Summary statistics, sub sample of banks for which is optimal to issue CoCos

Variables	N	Mean	St. Dev.	Min	Median	Max
leverage_actual_2a	5,344	0.141	0.065	0.033	0.129	0.236
Franchise_overMVA_2a	5,344	0.227	0.178	0.000	0.190	0.623
Optimal_vol_2a	5,344	0.040	0.027	0.001	0.035	0.084
Optimal_leverage_2a	5,344	0.178	0.090	0.034	0.154	0.317
Optimal_policyrate_2a	5,344	0.116	0.104	0.000	0.111	0.330
CoCos_overTotalExposure	5,344	0.032	0.049	0.000	0.006	0.245

In the next table, we show the summary statistics for the key figures of the sub sample of banks for which is not optimal to issue CoCos.

Table 12: Summary statistics, sub sample of banks for which is not optimal to issue CoCos

Variables	N	Mean	St. Dev.	Min	Median	Max
leverage_actual_2b	3,896	0.121	0.023	0.024	0.119	0.236
Franchise_overMVA_2b	3,896	0.152	0.139	0.000	0.110	0.408
Optimal_vol_2b	3,896	0.035	0.011	0.011	0.033	0.056
Optimal_leverage_2b	3,896	0.120	0.047	0.023	0.120	0.213
Optimal_policyrate_2b	3,896	0.202	0.068	0.062	0.210	0.330

In the appendix we show the summary statistics and the confidence intervals for the key optimized variables. The following two tables reports the value for clusters $2a1$ and $2b1$.

Table 13: Summary statistics, subsample of banks for which is optimal to issue CoCos, cluster 2a1

Variables	N	Mean	St. Dev.	Min	Median	Max
leverage_actual_2a1	907	0.159	0.059	0.035	0.155	0.236
Franchise_overMVA_2a1	907	0.131	0.126	0.00000	0.092	0.357
Optimal_vol_2a1	907	0.064	0.048	0.0001	0.049	0.133
Optimal_leverage_2a1	907	0.204	0.089	0.039	0.208	0.371
Optimal_policyrate_2a1	907	0.079	0.110	-0.050	0.053	0.330
CoCos_overTotalExposure2a1	907	0.035	0.046	1.0e-08	0.022	0.240

Table 14: Summary statistics, subsample of banks for which is not optimal to issue CoCos, cluster 2b1

Statistic	N	Mean	St. Dev.	Min	Median	Max
leverage_actual_2b1	900	0.148	0.039	0.038	0.145	0.236
Franchise_overMVA_2b1	900	0.064	0.063	0.000	0.044	0.173
Optimal_vol_2b1	900	0.037	0.011	0.013	0.036	0.059
Optimal_leverage_2b1	900	0.147	0.037	0.060	0.144	0.215
Optimal_policyrate_2b1	900	0.175	0.070	0.032	0.180	0.328

The subsequent tables show the results for clusters $2a2$ and $2b2$.

Table 15: Summary statistics, subsample of banks for which is optimal to issue CoCos, cluster 2a2

Variables	N	Mean	St. Dev.	Min	Median	Max
leverage_actual_2a2	2,075	0.132	0.054	0.035	0.122	0.236
Franchise_overMVA_2a2	2,075	0.171	0.137	0.00000	0.138	0.431
Optimal_vol_2a2	2,075	0.043	0.029	0.0001	0.039	0.092
Optimal_leverage_2a2	2,075	0.158	0.067	0.037	0.140	0.270
Optimal_policyrate_2a2	2,075	0.105	0.099	−0.050	0.092	0.304
CoCos_overTotalExposure2a2	2,075	0.027	0.048	1.0e-08	0.008	0.242

Table 16: Summary statistics, subsample of banks for which is not optimal to issue CoCos, cluster 2b2

Variables	N	Mean	St. Dev.	Min	Median	Max
leverage_actual_2b2	2,000	0.127	0.043	0.034	0.126	0.236
Franchise_overMVA_2b2	2,000	0.131	0.118	0.000	0.099	0.356
Optimal_vol_2b2	2,000	0.036	0.010	0.015	0.035	0.056
Optimal_leverage_2b2	2,000	0.126	0.042	0.033	0.126	0.207
Optimal_policyrate_2b2	2,000	0.198	0.060	0.081	0.200	0.319

This last set of two tables present the results for clusters $2a3$ and $2b3$.

Table 17: Summary statistics, subsample of banks for which is optimal to issue CoCos, cluster 2a3

Variables	N	Mean	St. Dev.	Min	Median	Max
leverage_actual_2a3	2,362	0.142	0.074	0.033	0.125	0.236
Franchise_overMVA_2a3	2,362	0.305	0.183	0.00000	0.330	0.833
Optimal_vol_2a3	2,362	0.032	0.020	0.0001	0.029	0.064
Optimal_leverage_2a3	2,362	0.203	0.127	0.034	0.173	0.379
Optimal_policyrate_2a3	2,362	0.139	0.098	−0.050	0.143	0.330
CoCos_overTotalExposure2a3	2,362	0.035	0.049	1.0e-08	0.016	0.245

Table 18: Summary statistics, subsample of banks for which is not optimal to issue CoCos, cluster 2b3

Statistic	N	Mean	St. Dev.	Min	Median	Max
leverage_actual_2b3	996	0.083	0.042	0.024	0.071	0.236
Franchise_overMVA_2b3	996	0.273	0.161	0.000	0.285	0.693
Optimal_vol_2b3	996	0.028	0.010	0.009	0.027	0.046
Optimal_leverage_2b3	996	0.078	0.031	0.023	0.071	0.133
Optimal_policyrate_2b3	996	0.232	0.072	0.072	0.250	0.330

The following table shows the confidence intervals for the key optimized variables, taking into account a confidence interval at 95% and quantiles are taken from the normal distribution given the high number of observation and the underlying framework. Confidence intervals for cluster 21

Table 19: Confidence Intervals for key optimized variables, confidence level 95%

Variables	Left hand endpoint	Right hand endpoint
Franchise_overMVA_2a	0.2219	0.2315
Optimal_vol_2a	0.0392	0.0406
Optimal_leverage_2a	0.1758	0.1806
Optimal_policyrate_2a	0.1129	0.1185
CoCos_overTotalExposure	0.0310	0.0336
Franchise_overMVA_2b	0.1474	0.1561
Optimal_vol_2b	0.0343	0.0350
Optimal_leverage_2b	0.1185	0.1214
Optimal_policyrate_2b	0.1997	0.2039

Confidence intervals for cluster 22. Confidence intervals for cluster 23.

Table 20: Confidence Intervals for key optimized variables, cluster 21

Variables	Left hand endpoint	Right hand endpoint
Franchise_overMVA_2a1	0.1224	0.1389
Optimal_vol_2a1	0.0606	0.0669
Optimal_leverage_2a1	0.1985	0.2100
Optimal_policyrate_2a1	0.0718	0.0861
CoCos_overTotalExposure2a1	0.0321	0.0381
Franchise_overMVA_2b1	0.0598	0.0680
Optimal_vol_2b1	0.0364	0.0379
Optimal_leverage_2b1	0.1441	0.1490
Optimal_policyrate_2b1	0.1707	0.1799

Table 21: Confidence Intervals for key optimized variables, cluster 22

Variables	Left hand endpoint	Right hand endpoint
Franchise_overMVA_2a2	0.1651	0.1769
Optimal_vol_2a2	0.0416	0.0441
Optimal_leverage_2a2	0.1549	0.1607
Optimal_policyrate_2a2	0.1006	0.1091
CoCos_overTotalExposure2a2	0.0254	0.0296
Franchise_overMVA_2b2	0.1255	0.1359
Optimal_vol_2b2	0.0360	0.0369
Optimal_leverage_2b2	0.1245	0.1282
Optimal_policyrate_2b2	0.1959	0.2011

Table 22: Confidence Intervals for key optimized variables, cluster 23

Variables	Left hand endpoint	Right hand endpoint
Franchise_overMVA_2a3	0.2974	0.3121
Optimal_vol_2a3	0.0313	0.0329
Optimal_leverage_2a3	0.1979	0.2081
Optimal_policyrate_2a3	0.1347	0.1427
CoCos_overTotalExposure2a3	0.0335	0.0374
Franchise_overMVA_2b3	0.2626	0.2825
Optimal_vol_2b3	0.0277	0.0289
Optimal_leverage_2b3	0.0762	0.0801
Optimal_policyrate_2b3	0.2272	0.2361

Evolution of Tier 1 and total exposure cluster 2a3

It is interesting to see the evolution of Tier 1 and the total exposure for *cluster2a3*. We show that from one side the Tier 1 has increased sharply in the last period but on the other side the total exposure is depressed. This may be explained through an issue of Tier 1 instruments, which not being hybrid capital, it should be equity. This equity issue may have depressed the market value of the bank and thus the market value of the assets and the related franchise value.

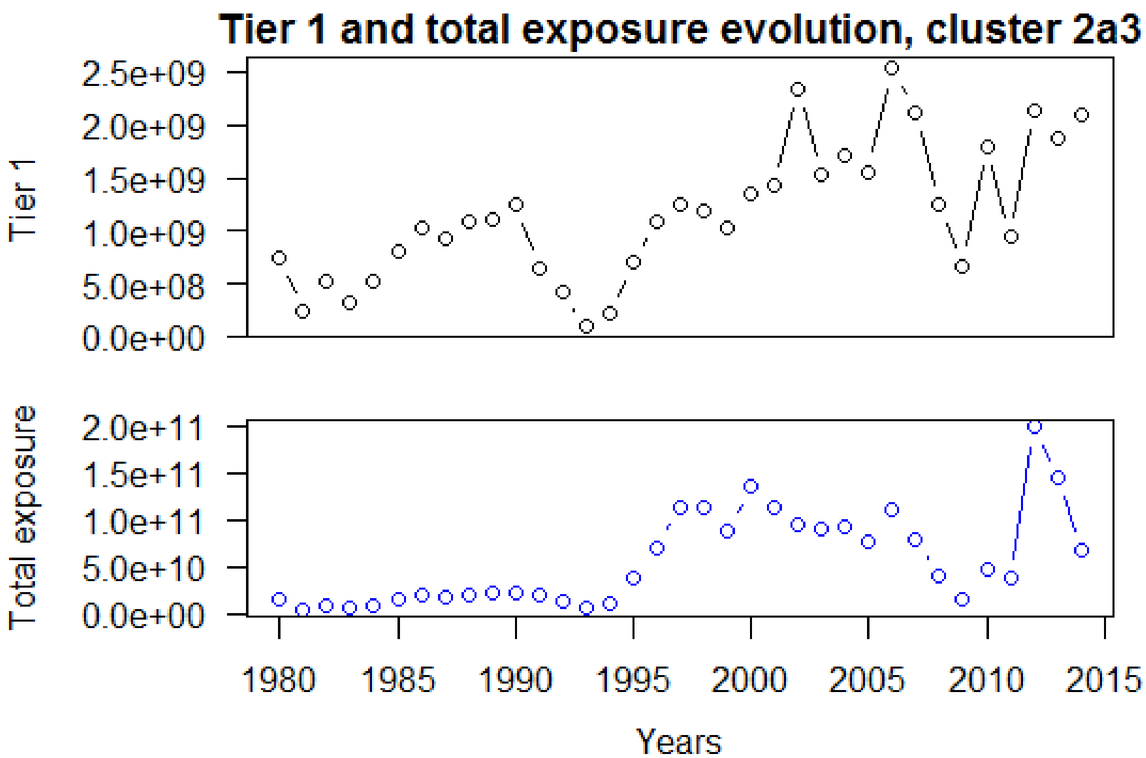


Figure 16: Tier 1 and total exposure evolution cluster 2a3.

Chapter 3

CoCo bonds and Write Down bonds impact on banks' risk appetite and investment policy[45]

Abstract

This paper investigates the risk appetite of a bank financed also with contingent convertible (CoCo) bonds and Write Down (WD) bonds and the impact this financing has on the investment policy. We conduct an empirical analysis on the whole spectrum of CoCos and WDs available over the time span 2009 - September 2015. We argue that the impact of those hybrid instruments on the bank investment policy is non-monotonic. It is described by a U-shaped curve for bank riskiness and by a inverted-U curve for bank growth. We show that issuing up to 25 bps of AD Tier 1 hybrids over total assets for the overall sample (up to 38 bps, considering the non systemic bank sample) is correlated with lower bank riskiness and higher growth opportunities. In general, there is still room for issuing hybrids belonging to AD Tier 1, containing bank riskiness and promoting bank growth but the marginal impact is not statistically significant for systemic banks. Tier 2 instruments outstanding are too many and have increased bank riskiness and decreased bank growth. We find similar results implementing a different model. Our results are, thus, relevant for investors, financial decision-makers, and regulators.

Keywords: risk appetite, growth opportunities, impact, Tier 1 hybrids, Tier 2 hybrids

[45]We acknowledge the financial support of the Europlace Institute of Finance (EIF) and the Labex Louis Bachelier (Grant 2016).

This paper proceeds as follows. In Section 13, we state the main objectives and explain the relevance of our research in the context of the existing literature. In Section 16, we present the research methodology and Section 18 shows the results. Finally, we conclude.

13 Introduction

The main goal of our research is to assess empirically banks' risk appetite and investment policy when their financing structure includes contingent convertible (CoCo) bonds and (or) write-down (WD) bonds, hybrid capital securities that absorb losses when the capital of the issuing bank falls below a certain threshold. The main difference between the two financing instruments is that CoCos convert to equity, while WDs convert to zero. We aim to study pros and cons of issuing WDs/CoCos with respect to their impact on banks' risk-taking and investments. To the best of our knowledge, this is the first attempt to address these questions empirically. In the post- Lehman Brothers failure, governments announced the end of the "too big to fail". In this context, issuing loss absorbing instruments has gained increasing popularity. Between 2009 and 2015, banks issued more than 380 billion of CoCos[46]. Regulation plays a crucial role in determining CoCos issuance. Under Basel III, CoCo bonds are eligible as either Additional Tier 1 (AT1) or Tier 2 (T2), which are types of capital apparently preferred by banks with respect to equity to accomplish regulatory requirements, given that they are cheaper and less dilutive than issuing equity. From 2009 to 2014, about half of the CoCos outstanding were eligible as AT1 and in 2015 about 76% were AT1 CoCos. The greatest amount of CoCos issued worldwide is in Europe, followed by Asia. In the US, banks have not yet issued this kind of instrument for many reasons: first, while implementing the Basel III, the US regulators does not allow CoCos to be part of the AT1 capital; second, the uncertainty related to tax deductibility of interest payments on CoCos is not resolved (von Furstenberg (2014)). Hence, Europe is the appropriate environment to study CoCos and WDs issues. The conclusions of this study are relevant for both the US and European regulators, financial decision-makers and investors. There is no convergence in the opinions concerning those instruments mainly due to the uncertainty around their impact and the difficulties in understanding their conversion mechanisms. There are CoCos supporters, like Switzerland's FINMA and Bank of England, and opponents, such as Deutsche Bank[47]. Our analysis is driven by the concerns economists have regarding the impact of CoCos and WDs on bank risk appetite. Neel Kashkari, Minneapolis Fed's President, warns against instruments like CoCos, suggesting that "..transferring risk to investors won't protect taxpayers from

[46]Data from Moody's Investors Service, Moody's Quarterly Rated and Tracked CoCo Monitor Database-Year End 2015

[47]Financial Times, ECB is having second thoughts on CoCo bonds, 04.24.2016

bailout"[48]. What is the impact of introducing hybrid capital like CoCos and WDs into banks' the financing structure on their risk appetite and investment decisions? In the time span considered, banks increased their Tier 1, issuing CoCos and WDs, and it is not clear whether they shift to safer assets in the period considered as the RWAs do not move a lot. We understand that we have a loss of information assessing the sample as a whole, thus we distinguish between systemic and non systemic banks, considering as systemic the largest 25%. Assessing the impact on bank riskiness and growth opportunities, what makes the difference among hybrids is the Tier to which they belong and not if they are WDs or CoCos. We find that the relation between hybrids and bank riskiness is non monotonic and can be described by a U-shaped curve. Interestingly, the relation between bank growth opportunities and hybrids is also non monotonic but we display a inverted-U shaped curve. We show that issuing up to 25 bps of AD Tier 1 hybrids over total assets for the overall sample (up to 38 bps, considering the non systemic bank sample), is correlated with lower bank riskiness and higher growth opportunities. Up to those figures, also the marginal impact is significant. For systemic banks it is also true that there is still room for issuing hybrids belonging to AD Tier 1, containing bank riskiness and promoting bank growth but the marginal impact is not statistically significant. Shifting our focus on Tier 2 instruments, we find that there are too many of them outstanding, meaning that with the actual amount present in the banking financing structure, the banks are increasing their riskiness and depressing their growth potential. The optimal amount of those kind of instruments is very small (less than 10 bps of Tier 2 hybrids relative to total assets). In the case of growth opportunities the optimal quantity is even smaller leading to display only the decreasing side of the parabola describing this behaviour. In the robustness check section, we propose a comparison of those results with an application of Aquila and Barone-Adesi (2017)'s model to this dataset. We show that on average it would be optimal to have 1.7% of AD Tier 1 hybrids but keeping a relative low volatility. This would be more in line with what the Basel regulators suggest proposing its capital requirement ratio in Basel III rule. In theory, the incentives go in the direction of the optimization results. From one side, there is an incentive in increasing Tier 1, and, on the other side, in decreasing RWAs.

14 Practical value

Currently, both academic and professional opinions disagree on the potential impact of CoCo and WD bonds on banks' financing and investing policies. Our ambition is to shed light on the potential side effects arising from including those instruments in the financing structure of banks. The results

[48]The Wall Street Journal, Fed's Kashkari Says Transferring Risk to Investors Won't Protect Taxpayers From Bailout, 04.18.2016

of this study should be of major interest for regulators and financial decision makers. Additionally, understanding and assessing the degree of risk-shifting incentives faced by banks is important for various types of investors (banks' shareholders and debt-holders), but also to the broader class of investors in those instruments.

15 Literature review

We rely on two main strands of literature on contingent convertible bonds. Given our research interest, first, we focus on the papers assessing the riskiness of banks and their propensity to risk-shift, second, on those assessing agency issues. Hilscher and Raviv (2014) argue that CoCos are an effective tool for stabilizing financial institutions. They find the optimal conversion ratio that eliminates stockholders' incentives to risk-shift. A first comprehensive empirical study on CoCos is conducted by Avdjiev et al. (2015). They interestingly show that CoCos issuance reduces banks' credit risk and investors in CoCos view those instruments as risky and place a significant likelihood on the possibility of conversion.The conversion trigger is widely assessed in the literature. Flannery (2010) proposes that the trigger be based on the market value of equity. Sundaresan and Wang (2015) illustrate that a market value –based conversion trigger for contingent convertibles may lead to multiple equilibrium and market manipulations. A unique equilibrium is reached only in the case in which there is no value transfer between bank equity and contingent debt at conversion. Wealth transfer among different categories of stakeholders is relevant for governments. Roy and El-Herraoui (2016) demonstrate the complexity of designing a fair and effective bail-in regime. The regulator is mainly confronted with the choice of implementing or not the wealth transfer. If it chooses to do so, it faces the risk of requests for compensation and arbitrage behaviour in financial markets. Maes and Schoutens (2012) discuss the impact of the trigger event and the conversion ratio on hazard contagion and the death spiral of systemic risk. Attaoui and Poncet (2015) develop the model showing that credit spread on straight debt is lower if the firm has WD bonds in its financing structure, given the cushion function of the WDs with respect to the straight debt (senior). CoCos are nearer to equity because in some states of the world they are not debt. Converting to equity, agency costs of equity are greater for CoCos with respect to write-downs that never transform into equity. Thus write-downs are more efficient in solving agency costs of equity. Both of them diminish debt overhang problem and the bankruptcy costs (the present value of bankruptcy costs). Furthermore, WDs have some advantages with respect to CoCos: there is no multiple equilibrium, no market manipulation (to avoid losses), that at least partially solves the death spiral issue. On the bankruptcy costs side, Chen et al. (2013) show that replacing some straight debt with CoCos lowers the endogenous default barrier and therefore increases the firm's ability to mitigate a loss in asset value. This reduces bankruptcy costs and increases the value of equity. Albul et al. (2010) provide a tax shield argument in favour of the hybrid capital. They conclude that banks should substitute CoCos for straight debt because, on one hand, they provide the same protection as equity, and, on the other hand, they allow the most of the tax shield benefits. Different governments do not allow the same tax treatment of the hybrid capital, thus differences in tax shield could be relevant driver of the preference to issue WDs over CoCos.

16 Main hypothesis development

16.1 CoCos, WDs and banks' risk-taking behavior

Banks started issuing CoCos in the late 2009, after the enforcement of Basel III rule (even if its full application starts in 2015), whose focus on capital requirements deals with the ratio between RWA and Tier1 which has to be larger than 6% a predetermined threshold. In this perspective an issue of Tier 1 instruments should decrease the ratio (enlarging the denominator) per se even if keeping RWA constant. Considering the potential impact of a bond converted to equity, we expect that the existing shareholders would resist to engage in excessive risk-taking projects because otherwise they would share the bank capital with the new class of "converted" equity-holders. In this spirit we argue ex-ante that the CoCos of both Tier1 and Tier2 should decrease the riskiness of the assets. The fact that Basel III allows the banks to issue only small amount of hybrids and the literature concerning the death-spiral, lead us to think that there might be a non-monotonic relation among the hybrids outstanding and the riskiness of the assets.

Hypothesis I: For smaller amount of CoCos and WDs issued, the bank riskiness decreases and the relation is non-monotonic.

We consider the following OLS fixed-effects (within) regression, where we want to understand the non-monotonic relation between the hybrid instruments and the riskiness of the bank. We do not distinguish between CoCos and WDs, since our focus relies on the difference between Tier1 and Tier2 instruments.

$$RWAA_{i,t} = \alpha + \beta_1 ADTier1_{i,t} + \beta_2 Tier2_{i,t} + \beta_3 ADTier12_{i,t} + \beta_4 Tier22_{i,t} + \gamma Controls_{i,t} + \delta_i + \tau_t + v_{i,t} \quad (42)$$

Where $RWAA$ is our proxy for the bank riskiness. We smooth the bank RWA, assessing it relative to the total assets (book value). The explaining variables are (i) $ADTier1$, the outstanding amount of CoCos and WDs belonging to Tier 1 over total assets, (ii) $Tier2$, the outstanding amount of CoCos and WDs belonging to Tier 2 over total assets, (iii) $ADTier12$, the square of the outstanding amount of CoCos and WDs belonging to Tier 1 over total assets, (iv) $Tier22$, the square of the outstanding amount of CoCos and WDs belonging to Tier 2 over total assets. Those last two elements capture the non-monotonic relation. t indexes calendar quarters, $ADTier1_{i,t}$ and $Tier2_{i,t}$ represent the amount outstanding for bank i in quarter t of CoCo bonds and WD bonds as a fraction of total assets. We perform the same regression also per year and we observe similar results. δ_i are bank fixed-effects and τ_t are time fixed effects. $Controls_{i,t}$ is a vector of time-varying bank characteristics (for the while we display the results only for leverage, but we performed the regression controlling also for the logarithm of the total assets and results do not change).

Before conversion, we could expect that the two different Tier instruments might impact differently over the risk appetite of the bank, thinking about it in a Merton framework, where Additional Tier 1 instruments should decrease the default put option and Tier 2 hybrids should increase it. After conversion, instead, they both accrue to the Common equity Tier 1 diminishing the default put. We expect that it is their contingent convertibility the key argument having an impact in shaping the risk appetite of the bank. In a scenario where conversion is not such a remote event, the bank should not be encouraged to undertake too risky projects that would lead to a conversion, without distinguishing between ADT1 or Tier 2 instruments. Hence, we expect to have β_1 and β_2 smaller than zero. β_3 and β_4, the beta of the quadratic terms should be of the opposite sign, due to the fact that we expect that after a certain threshold the behaviour might change. We assess for each kind of hybrids, where is the minimum of the parabola describing the behaviour of the $RWAA$ with respect to each Tier hybrids instruments. First, we assess the impact those hybrids instruments have on the bank riskiness for the whole sample we have at disposal. Then we divide it into systemic and not systemic banks, considering as systemic, the banks with total assets in the top 25%. This lead us to two other related hypothesis.

Hypothesis II: For non systemic banks, smaller quantities of hybrid capital issued belonging to Tier 1 play a role in decreasing the bank riskiness and the relation is non-monotonic.

Hypothesis III: Systemic banks have issued already too many CoCos and WDs, thus any further issue increase the bank riskiness and the relation is non-monotonic.

For systemic banks, we expect that the average value issued of hybrids to be nearer to the minimum of the parabola or it might be even larger. CoCos and WDs have been first issued by larger banks also because of the appeal they had for improving the regulatory requirements figures without modifying the investment strategy and consequently the composition of RWAs.

16.2 CoCos, WDs and banks' growth opportunities

On one side the bank has to balance its riskiness, also to accomplish to the Basel requirements. On the other side, the bank has to prepare a capital structure which is able to sustain also growth opportunities. As we did above, we do not distinguish between CoCos and WDs, since our focus relies on the difference between Tier1 and Tier2 instruments. *Ceteris paribus*, the hybrids contribute to decrease the bank riskiness and promote the growth opportunities. We test this hypothesis on the whole sample.

Hypothesis IV: For smaller amount of CoCos and WDs issued, the bank growth opportunities increase and the relation is non-monotonic.

We propose the following OLS fixed-effects (within) regression, in order to assess the non-monotonic relation between the hybrid instruments and the banks' growth opportunities.

$$Q_{i,t} = \alpha + \beta_1 ADTier1_{i,t} + \beta_2 Tier2_{i,t} + \beta_3 ADTier12_{i,t} + \beta_4 Tier22_{i,t} + \gamma Controls_{i,t} + \delta_i + \tau_t + v_{i,t} \quad (43)$$

Where Q is the Q ratio, defined as the ratio between market value of the bank its book value, and is our proxy for the banks' growth opportunities. Given that we want to understand the optimal capital structure for the banks in our sample, we do not modify the explanatory side of the equation. We expect that the hybrids contribute to increase the Q ratio of the bank being resources available for being invested in new projects. Our argument relies also in this case on the contingent convertibility of those instruments. Focusing on this element, there is a clear implied incentive for the bank not to undertake too risky projects otherwise the present shareholders' wealth might be impaired by the consequent dilution due to the conversion of the hybrids. In this case, β_1 and β_2 should be positive and, conforming to the non-monotonic relation, β_3 and β_4 should be negative. We argue that for smaller amount of hybrids issued there is no significant capital dilution, but for larger amount of them there are too many fresh financial resources available that there might be the incentive to invest in value-destroying projects. Similarly, we assess this issue in a framework where we describe the growth opportunities with a Down-and-Out call (DOC) option (as in Aquila and Barone-Adesi (2017), please refer to the model summarized in the Appendix). On one hand, for smaller quantity of hybrids belonging to AD Tier 1 and for relative lower volatility, the DOC option increases in value because the distance to the barrier is increased and the probability to touch the barrier is relative smaller. On the other hand, issuing too many hybrids might be correlated with higher bank riskiness which might lead to the touch of the barrier and the consequent expiration of the growth opportunities. Those elements drive us to test the following two hypothesis.

Hypothesis V: For non systemic banks, smaller quantities of hybrid capital issued belonging to Tier 1 play a role in increasing the bank's growth opportunities and the relation is non-monotonic.

Hypothesis VI: Systemic banks have issued already too many CoCos and WDs, thus any further issue decrease the bank's growth opportunities and the relation is non-monotonic.

We expect that too many hybrids may lead to death spiral or other issues discussed in the literature which could damage growth opportunities. From a methodological point of view, we contribute also in promoting a new way of interpreting the regression results. Thanks to our analysis concerning the marginal impact, we understand more deeply the dynamic of the results obtained through the regression.

17 Data

We collect data on CoCos and WD bonds issued worldwide from Bloomberg. These kind of issues are available since 2009. Since we distinguish between Tier 1 and Tier 2, we consider only hybrids clearly attributable to Tier 1 or Tier 2. We get a historical data on the Risk-Weighted Assets (RWAs) of the issuing banks and their issue characteristics. Relevant issue characteristics include: identifiers of the issuer (CUSIP, ISIN, name, Bloomberg Ticker); country of the issuer; issue date; maturity date; issue type, i.e., whether the issued bond is a CoCo or WD bond; trigger level; amount issued; tier type and rating; balance sheet items and market prices. Assessing the summary statistics of the data (Table 23), in a time series perspective, we observe the following key stylized facts. First of all, the median of the ratio between Tier 1 and the risk-weighted assets double itself in the time span considered (Figure 17). This is mainly due to an increase in the Additional Tier 1 which is part of the broader Tier 1 (Figure 18) as it easy to understand the relative preference in issuing other instruments than equity. Second, the risk-weighted assets do not sink at all during after 2009 (Figure 19), they do not move a lot, thus the improvement in the ratio is mainly driven by the increase in the Additional Tier 1. This is interesting given that Basel III focusing on the ratio between Tier 1 and RWAs, and giving it a lower bound of 6%[49], gives the bank two channel for improving it: from one side, the bank might increase the Tier1, and from the other side, the bank could decrease the RWAs. We remark that the banks in the sample increase the Additional Tier 1 and do not diminish the RWAs. Given the period considered, it is not too difficult to understand that the growth opportunities, identified through the Q ratio, swing. They start recovering in the second half of 2012 (Figure 20).

18 Results

18.1 Do CoCos and WDs play a role in decreasing banks' riskiness?

This section presents the main results concerning the first three hypothesis. The results are reported in Table 24. Column (1) display the results for *Hypothesis I* and the other two columns show the results of the test over *Hypothesis II and III*. Overall, we find that our results are consistent with our hypothesis. Starting from the regression table, we observe that in all the three regressions the *beta* of the first order variables are negative and the *beta* of the squared variables are positive. These results show that the relation between the RWAA and the explanatory variables can be described via a U-shaped curve, thus the hybrids outstanding first play a role in decreasing banks' riskiness and after reaching the minimum level banks' riskiness increases. Considering the *Hypothesis I*, Figure 21 exhibits that the marginal effect is negative and significant up to 53 bps. It means that, *ceteris paribus*, issuing up to 53 bps of hybrids (relative to total assets) belonging to AD Tier 1 is correlated with a decrease in the bank riskiness. After this threshold the impact is no more significant. Our banks have issued exactly 53 bps thus they are at the optimum in terms of relative amount of AD Tier1 instruments issued for decreasing RWAA. The minimum of the U-shaped curve is at 166 bps but after 53 bps we show that the impact on RWAA is not statistically significant. Regarding Tier 2 hybrids, it is interesting to see that it is the squared parameter which is significant in the regression table and Figure 22 shows that in this sample there are already too many Tier 2 hybrids. This means that we are already in the increasing side of the U-shaped curve where the bank riskiness increases with the amount outstanding of Tier 2 hybrids. We find that the minimum is at 5bps, thereafter the impact of the Tier 2 hybrids is statistically significant above 13bps. On average we have 10 bps of those hybrids outstanding meaning that we are already on the increasing side of the curve. Thus those hybrids are correlated with higher bank riskiness. Moving to the *Hypothesis II*, we focus on non systemic banks. At a first glance, we might think that for each 1% more of hybrids

[49]This lower bound has been applicable since 2015, thus we consider the cumulative amount of the hybrids at stake for each bank

outstanding the RWAA figure decrease by 0.08% if the hybrids belong to AD Tier 1 and by 0.55% if they are Tier 2; however, the marginal effect depends also on the squared term, the marginal effect is depicted in figure 24. For the AD Tier 1 CoCos and WDs this impact is also statistically significant but the magnitude of the increasing impact is smaller than in the case of Tier 2 hybrids (Table 24). The marginal impact of ADT1 instruments shows that on average we are at 63bps and the minimum of the parabola is at 151bps meaning that there is still room for decreasing RWAA issuing those hybrids. Anyway, we find that this figure might not be taken into account since the marginal impact is not statistically significant(Table 27, Table 28 and Figure 23). In the case of Tier 2 instruments, we find that the bank behaviour moved toward riskier assets, since the minimum of the parabola is reached for 4bps and the average issue consists of 14bps which is also the threshold above which the impact is considered to be significant (Table 27, Table 28 and Figure 24).

Hypothesis III is not confirmed in the data. Larger institutions have already too many hybrids outstanding (both AD Tier 1 and Tier 2) that the marginal impact of any change in the amount of issued hybrid capital would not affect the bank riskiness. In Figure 25, we can see that in this case we are in the increasing part of the parabola, but the marginal impact is not statistically significant.

18.2 Do CoCos and WDs impact positively banks' growth opportunities?

We obtain very interesting results regarding the impact that CoCos and WDs have on banks' growth opportunities. We find that the relation between growth opportunities and hybrids outstanding is non-monotonic and appears to be well described by a inverted-U shaped curve in conformity with *Hypothesis IV*. Hence, this is an opposite result with respect to what we find above. For smaller amount of hybrids outstanding, the impact over the banks' growth opportunities is positive, up to the maximum level reached by the curve, after which the impact of the hybrids starts depressing banks' growth. Looking at Table 31, we find results consistent with the hypothesis. β_1 and β_2 are positive and the beta of the squared variables are negative as predicted in *Hypothesis IV*.

The inverted-U shape of the *ceteris paribus* dynamic of the Q ratio with respect to a variation in the hybrids outstanding shows both its increasing and decreasing side only for AD Tier 1 instruments belonging to the overall sample and to the non systemic banks sub-sample Assessing the overall sample, the maximum of the parabola is reached for 220bps and on average our banks have issued 53 bps. We are still on the increasing side of the parabola and there should be room for issuing hybrids and still increase growth opportunities. Nevertheless, we find that this impact is significant only till 25bps (Table 32, Table 33, Figure 27 and Figure 28). This results are mainly driven by the sub-sample of non systemic banks. Those banks have issued on average 63bps of hybrids and the maximum of the parabola is reached for 227bps. Apparently, *Hypothesis V*, is satisfied because a further issue of hybrids could still be correlated with an increase banks' growth. However, the impact is significant only till 38bps of hybrids issued. For Tier 2 instruments in both cases the amount issued are much larger than the maximum of the parabola but none of the results are statistically significant. For the Q ratio, Tier 2 hybrids, before conversion, are more similar to debt and less to equity and this is captured by the fact that we can find only the decreasing side of the inverted-U curve for Tier 2 instruments. For systemic banks we find not statistically significant results, even if in general we obtain only the decreasing side of the parabola. Thus the results for this category of banks are not consistent with *Hypothesis VI*.

18.3 Robustness check

We have already discussed that our regressions are robust controlling for leverage, banks dimensions, separate specification of CoCos and WDs. More interestingly, we compare the empirical results we obtained above with what predicts the model developed by Aquila and Barone-Adesi (2017) applied to our sample. The model is described in the Appendix. In this model, the bank is assessed in a Merton framework where the manager maximizes the bank value, which is partly due to the sum of a down-and-out call option and a default put option. The optimization variables are leverage, and volatility. We obtain the optimal quantity of hybrids belonging to AD Tier 1 comparing the optimal leverage to the actual one, expressed in Basel terms, i.e. the ratio between Tier 1 and the total exposure. In this model we assimilate Tier 2 hybrids before conversion to debt. In this specification, the default put option is favoured by Tier 2 instruments. On the other side the DOC option is an increasing function of AD Tier 1 instruments and decreasing one of Tier 2 hybrids. We find that on average the banks in the sample should issue 1.7% of AD Tier 1 instruments, while keeping a 7.7% of optimal leverage and a 3.6% of volatility (Table 38 and Figure 33). In the sample there are also some banks who have already too many hybrids outstanding, which are the larger ones. We have to point out that the median optimal volatility is 2.5%. In this model, the optimal parameters comes from a joint optimization procedure. This is crucial in our analysis, because in Basel III, the regulator propose as capital requirements target ratio the one given by Tier 1 over RWAs. We show that for having an average 1.7% of AD Tier 1 instruments outstanding it is necessary to have a quite low volatility. We stress this important issue because we have seen that on average, in the sample span considered, the banks didn't decrease the RWAs at all, they kept them constant and the results of the regression we performed above signal that*ceteris paribus* there are too many hybrids outstanding, thus the RWAs are even increased and Q ratio depressed. This might be given also because the banks didn't shift their investment policy to safer assets.

19 Conclusions

In this paper, we want to understand the impact Additional Tier 1 CoCos and WDs and Tier 2 hybrid instruments have on the bank riskiness and growth opportunities. This is one of the few empirical papers on CoCos and WDs, due to the scarcity and bad quality of the data available. We test our hypothesis over all the data available from banks spread all over the world. In conformity to our hypothesis we find that (i) hybrids have a non monotonic impact on bank riskiness and on bank growth opportunities. Interestingly, on one side, the relation between hybrids and bank riskiness is described by a U-shaped curve and, on the other side, with respect to growth opportunities, we have a inverted-U shaped curve, leading to a trade-off. Hence, it is relevant to find a balance between containing bank riskiness and contemporaneously promoting bank growth. The banks in our sample issued an amount of AD Tier 1 hybrids which is correlated with a decrease in bank riskiness and an improvement in bank growth opportunities. We show that (ii) for a smaller amount of AD Tier1 hybrids outstanding (up to 25 bps for the overall sample and up to 38 for the non systemic banks), the impact is significant. (iii) From one side, it is correlated with lower bank riskiness, on the other side, with higher growth opportunities. Shifting our focus on systemic banks due to the sample composition we do not find any significant marginal impact but as in the previous analysis, the average issue is still far from the minimum (maximum) of the parabola for the bank riskiness (growth opportunities). (iv) Considering Tier 2 instruments, we find that there are too many of them outstanding and, in the case of bank riskiness, (v) our banks have overtaken the minimum of the parabola and place themselves in the increasing side promoting, this way, bank riskiness. Their marginal impact is also significant for the overall and non systemic bank sample. In the case of the Q ratio, Tier 2 instruments are also too many but the marginal impact is not statistically significant and the maximum of the parabola is reached later with respect to the minimum in the case of the bank riskiness. In the robustness check, we show also that (vi) on average it would be optimal to have on average 1.7% of AD Tier 1 hybrids but keeping a relative low volatility. This would be more in line with what the regulators suggest proposing its capital requirement ratio. In theory, the incentives go in the direction of the optimization results. From one side, there is an incentive in increasing Tier 1, and, on the other side, in decreasing RWAs.

Appendix

Tables and Figures

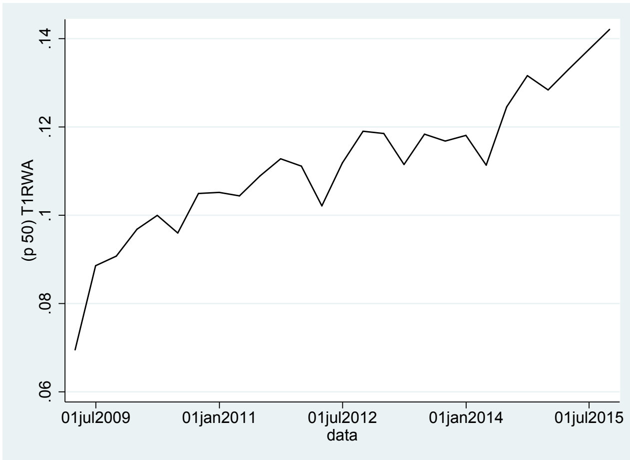


Figure 17: Tier1 ratio improvement between 2009 and 2015.

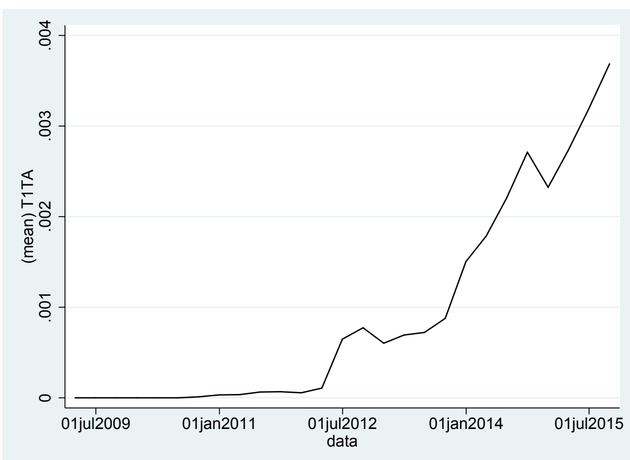


Figure 18: Additional Tier1 improvement between 2009 and 2015.

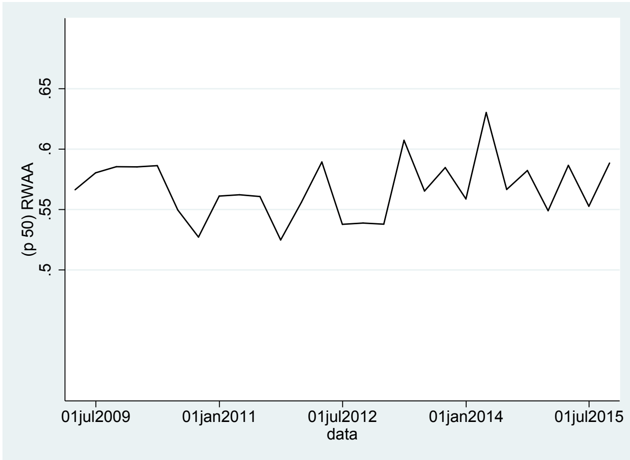


Figure 19: RWAs are constant between 2009 and 2015.

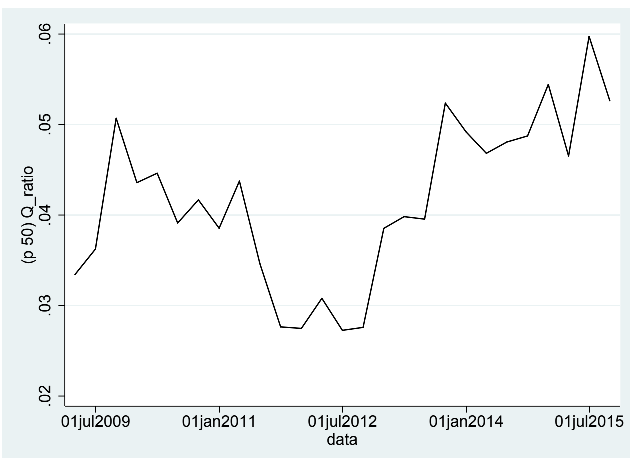


Figure 20: Q ratio swing.

Table 23: Summary statistics

Statistic	N	Mean	St. Dev.	Min	Median	Max
Tier1_TA	1135	0.050	0.023	0.008	0.048	0.145
ADT1_TA (in bps)	1135	0.010	0.124	0.000	0.000	3.226
T2_TA (in bps)	1135	0.0004	0.006	0.000	0.000	0.143
Q_ratio	1135	0.028	0.021	0.000	0.020	0.100
RWA_TA	1135	0.554	0.257	.000	0.566	5.483

Table 24: In this table, we assess the impact hybrids capital have on banks' riskiness, through the following regression: $RWAA_{i,t} = \alpha + \beta_1 ADTier1_{i,t} + \beta_2 Tier2_{i,t} + \beta_3 ADTier12_{i,t} + \beta_4 Tier22_{i,t} + \gamma Controls_{i,t} + \delta_i + \tau_t + v_{i,t}$. The regression has robust standard errors and includes controls for leverage, bank quarter fixed effects. In the first column it is performed on the whole sample, in the second column on the sub-sample of non systemic banks and in the third column we display results for the systemic sub-sample

	(1)	(Non Systemic)	(Systemic)
ADTier1	-0.1194	-0.0891	-0.3722
	(2.54)**	(1.71)*	(0.76)
Tier2	-0.7065	-0.5587	-7.7206
	(1.64)	(1.14)	(0.82)
ADTier12	0.0358	0.0294	0.9766
	(2.20)**	(1.69)*	(0.53)
Tier22	6.4170	5.8850	440.7535
	(3.81)***	(2.96)***	(0.71)
R-squared	0.05	0.06	0.16
N	1,140	857	283

Table 25: Tests: overall results of the hybrids impact over RWArelative to total assets

Statistic for	Testnl	chi2	Prob>chi2	Min (bps)	Not significant bef./aft.
ADT1 overall	$-\beta[ADTier1]/(2*\beta[ADTier12]) = 0$	43.93	0.000	166.76277	53
Tier2 overall	$-\beta[Tier2]/(2*\beta[Tier22]) = 0$	7.61	0.0058	5.5052165	13

Table 26: Mean estimation: mean Tier1

Statistic for	Mean	Std. Err.	Conf. intervals 95%	
Tier 1 if ADTier1>0	.5362476	.058966	.4201441	.6523511
Tier 2 if Tier2>0	.1087488	.0120724	.0845919	.1329057

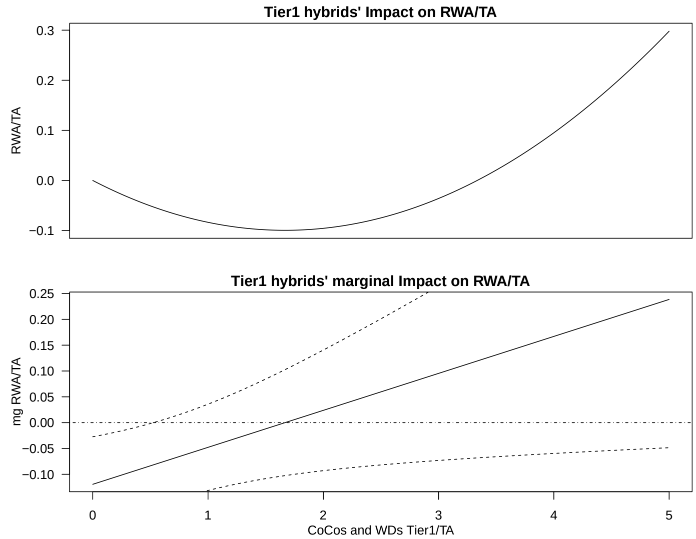


Figure 21: Tier1 hybrid instruments' impact over RWA relative to total assets.

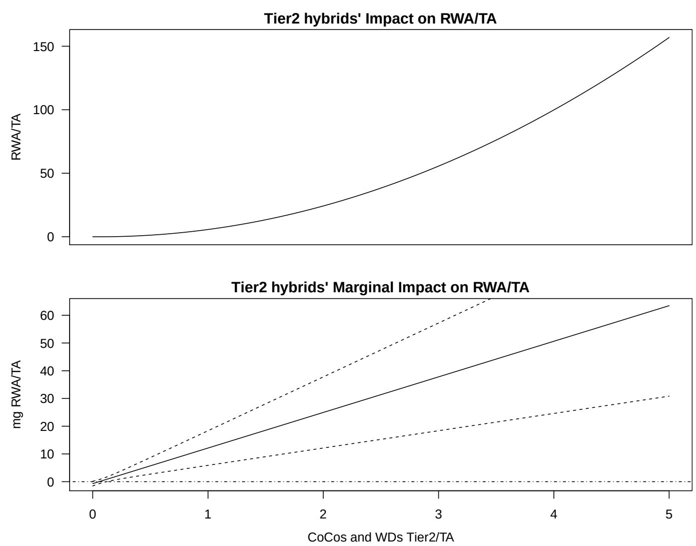


Figure 22: Tier2 hybrid instruments' impact over RWA relative to total assets.

Table 27: Non systemic banks: tests

Statistic for	Testnl	chi2	Prob>chi2	Min (bps)	Not significant bef./aft.
ADT1 non systemic	$-\beta[ADTier1]/(2*\beta[ADTier12])=0$	26.95	0.000	151.48119	NA
Tier2 non systemic	$-\beta[Tier2]/(2*\beta[Tier22])=0$	3.16	0.0755	4.7465829	14

Table 28: Non systemic banks: Mean estimation

Statistic for	Mean	Std. Err.	Conf. intervals 95%	
Tier 1 if ADTier1>0	.6311542	.0683327	.4964904	.7658181
Tier 2 if Tier2>0	.1420174	.0126398	.1165436	.1674913

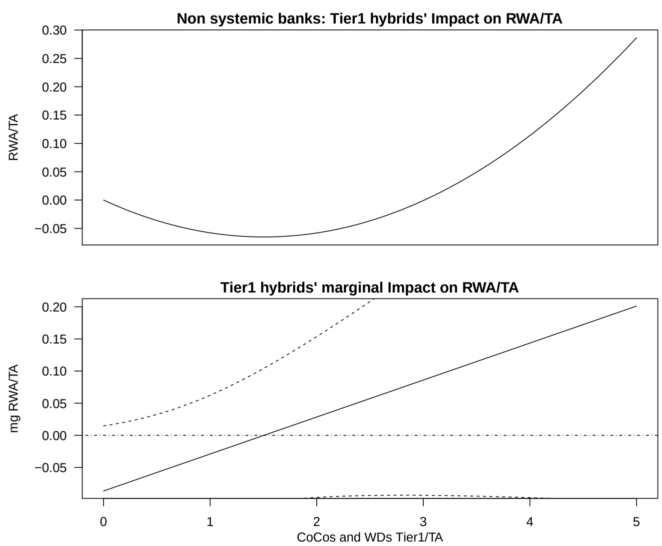


Figure 23: Non systemic banks: Tier1 hybrid instruments' impact over RWA relative to total assets.

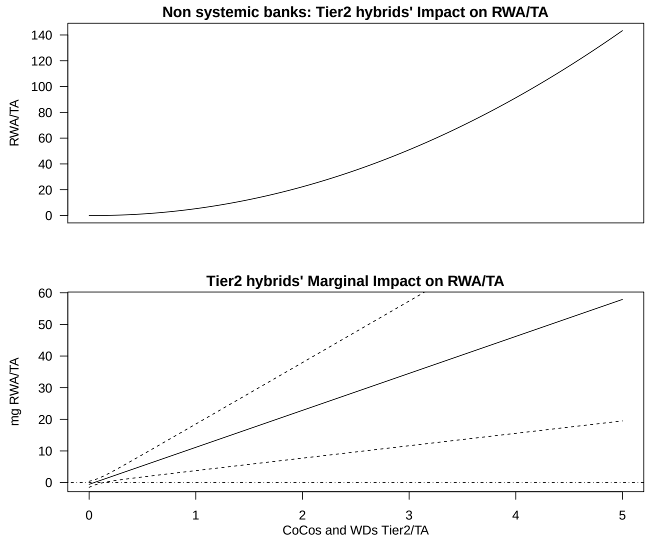


Figure 24: Non systemic banks: Tier2 hybrid instruments' impact over RWA relative to total assets.

Table 29: Systemic banks: tests

Statistic for	Testnl	chi2	Prob>chi2	Min (bps)	Not significant bef./aft.
ADT1 non systemic	$-\beta[ADTier1]/(2*\beta[ADTier12]) = 0$	2.63	0.1046	19.058562	NA
Tier2 non systemic	$-\beta[Tier2]/(2*\beta[Tier22]) = 0$	19.11	0.000	.87584093	NA

Table 30: Systemic banks: Mean estimation

Statistic for	Mean	Std. Err.	Conf. intervals 95%	
Tier 1 if ADTier1>0	.0489468	.0096185	.0295857	.068308
Tier 2 if Tier2>0	.0089428	.001337	.0060753	.0118103

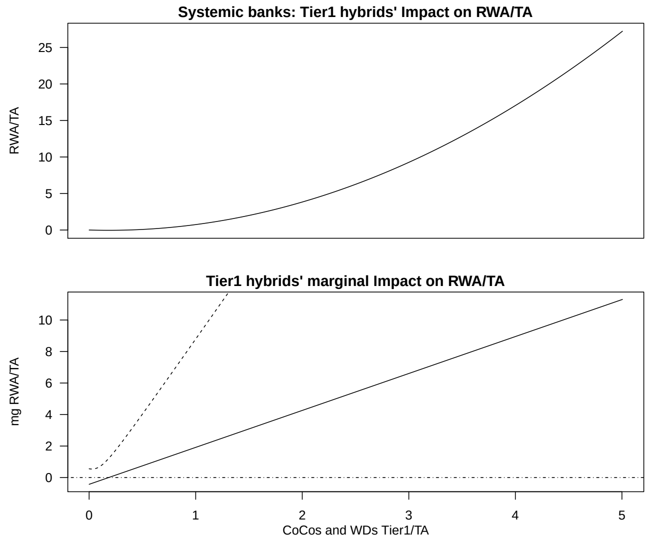


Figure 25: Systemic banks: Tier1 hybrid instruments' impact over RWA relative to total assets.

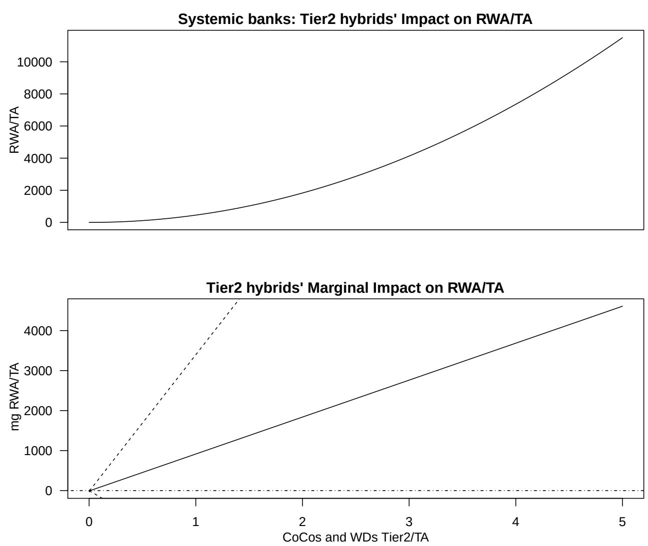


Figure 26: Systemic banks: Tier2 hybrid instruments' impact over RWA relative to total assets.

Table 31: In this table, we display the results the impact hybrids capital have on banks' growth opportunities. We use the Q ratio as our proxy for banks' growth opportunities and it is defined as the ratio between the market value of the assets and the book value. We perform the following regression: $Q_{i,t} = \alpha + \beta_1 ADTier1_{i,t} + \beta_2 Tier2_{i,t} + \beta_3 ADTier12_{i,t} + \beta_4 Tier22_{i,t} + \gamma Controls_{i,t} + \delta_i + \tau_t + v_{i,t}$. The regression has robust standard errors and includes controls for leverage, bank quarter fixed effects. In the first column it is performed on the whole sample, in the second column on the sub-sample of non systemic banks and in the third column we display results for the systemic sub-sample

	(1)	(Non Systemic)	(Systemic)
ADTier1	0.0359	0.0387	0.3333
	(2.09)**	(2.17)**	(1.42)
Tier2	0.1055	0.1713	-0.3740
	(0.60)	(0.85)	(0.13)
ADTier12	-0.0081	-0.0085	-1.1507
	(1.80)*	(1.89)*	(1.44)
Tier22	-0.6819	-0.9873	-82.5165
	(0.96)	(1.23)	(0.41)
R-squared	0.11	0.14	0.30
N	1,543	1,158	385

Table 32: Tests: overall results of the hybrids impact over Q ratio

Statistic for	Testnl	chi2	Prob>chi2	Min (bps)	Not significant bef./aft.
ADT1	$-\beta[ADTier1]/(2 * \beta[ADTier12]) = 0$	76.84	0.000	220.77335	25
Tier2	$-\beta[Tier2]/(2 * \beta[Tier22]) = 0$	2.24	0.1346	7.733651	NA

Table 33: Overall sample: Mean estimation

Statistic for	Mean	Std. Err.	Conf. intervals 95%	
Tier 1 if ADTier1>0	.529807	.0580228	.4155704	.6440436
Tier 2 if Tier2>0	.1087488	.0120724	.0845919	.1329057

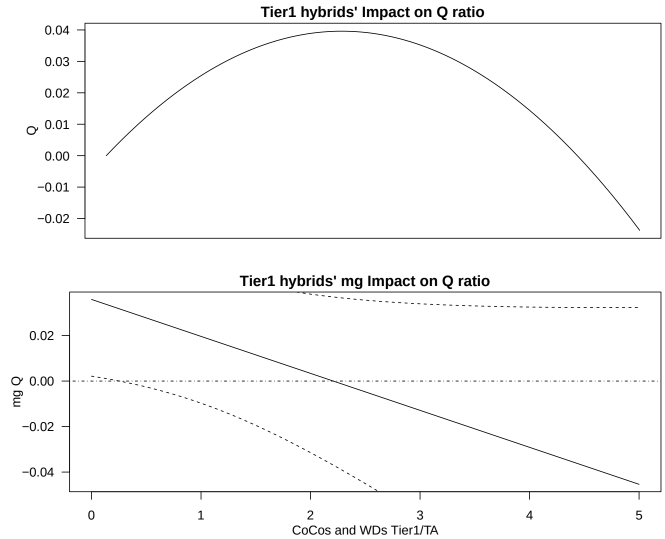


Figure 27: Tier1 hybrid instruments' impact over Q ratio.

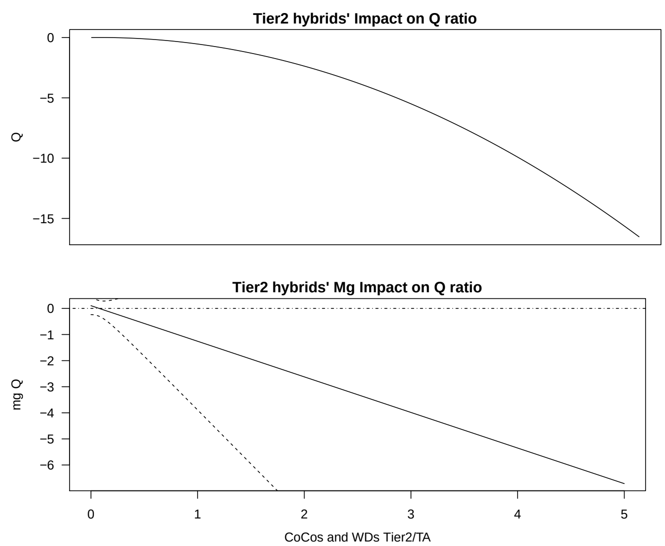


Figure 28: Tier2 hybrid instruments' impact over Q ratio.

Table 34: Tests for non systemic banks: results of the hybrids impact over Q ratio

Statistic for	Testnl	chi2	Prob>chi2	Min (bps)	Not significant bef./aft.
ADT1 non systemic	$-\beta[ADTier1]/(2*\beta[ADTier12]) = 0$	57.72	0.000	227.2684	38
Tier2 non systemic	$-\beta[Tier2]/(2*\beta[Tier22]) = 0$	6.13	0.0133	8.676156	NA

Table 35: Non Systemic banks: Mean estimation

Statistic for	Mean	Std. Err.	Conf. intervals 95%	
Tier 1 if ADTier1>0	.6311542	.0683327	.4964904	.7658181
Tier 2 if Tier2>0	.1420174	.0126398	.1165436	.1674913

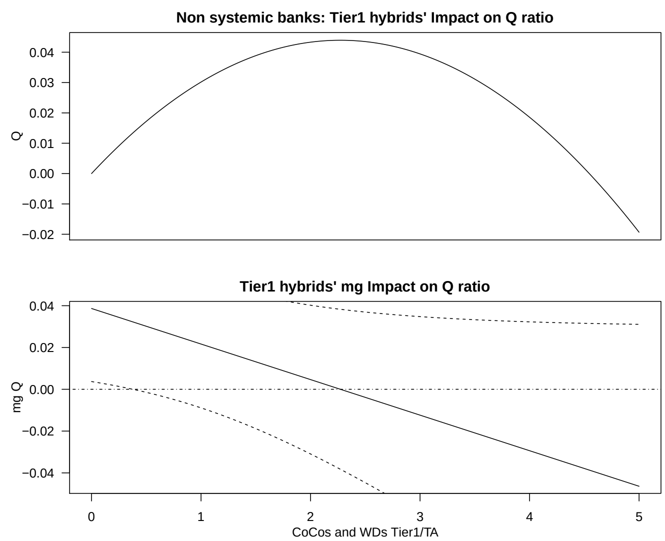


Figure 29: Non systemic banks: Tier1 hybrid instruments' impact over Q ratio.

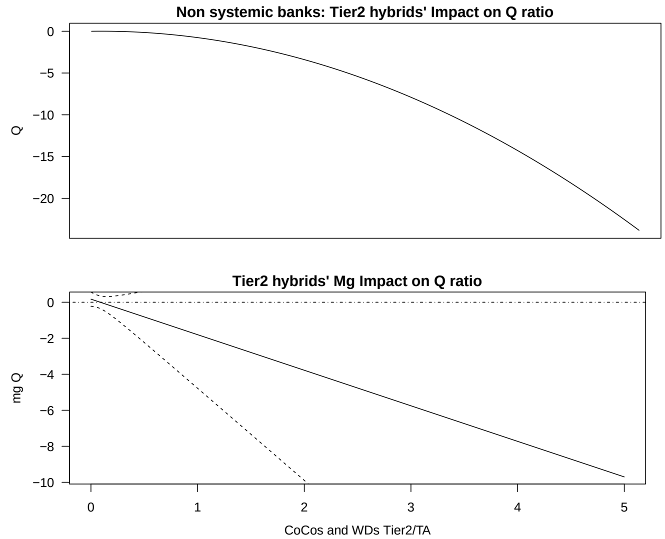


Figure 30: Non systemic banks: Tier2 hybrid instruments' impact over Q ratio.

Table 36: Tests for systemic banks: results of the hybrids impact over Q ratio

Statistic for	Testnl	chi2	Prob>chi2	Min (bps)	Not significant bef./aft.
ADT1 systemic	$-\beta[ADTier1]/(2*\beta[ADTier12])=0$	270.48	0.0000	14.481069	NA
Tier2 systemic	$-\beta[Tier2]/(2*\beta[Tier22])=0$	0.01	0.9191	-.22661568	NA

Table 37: Systemic banks: Mean estimation

Statistic for	Mean	Std. Err.	Conf. intervals 95%	
Tier 1 if ADTier1>0	.0489468	.0096185	.0295857	.068308
Tier 2 if Tier2>0	.0089428	.001337	.0060753	.0118103

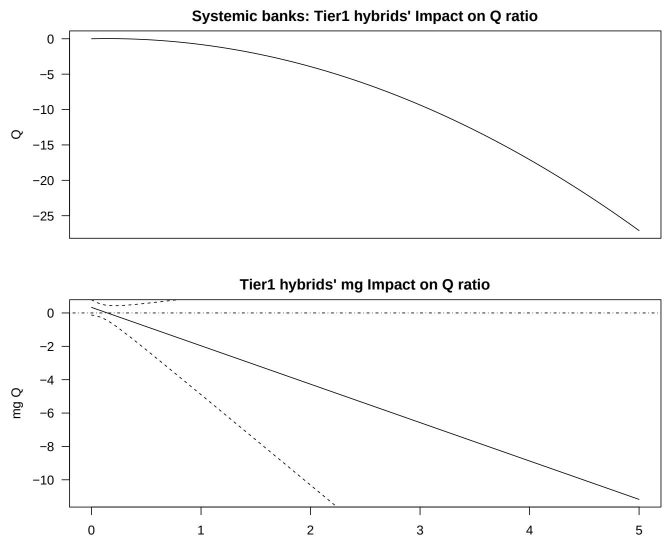


Figure 31: Systemic banks: Tier1 hybrid instruments' impact over Q ratio.

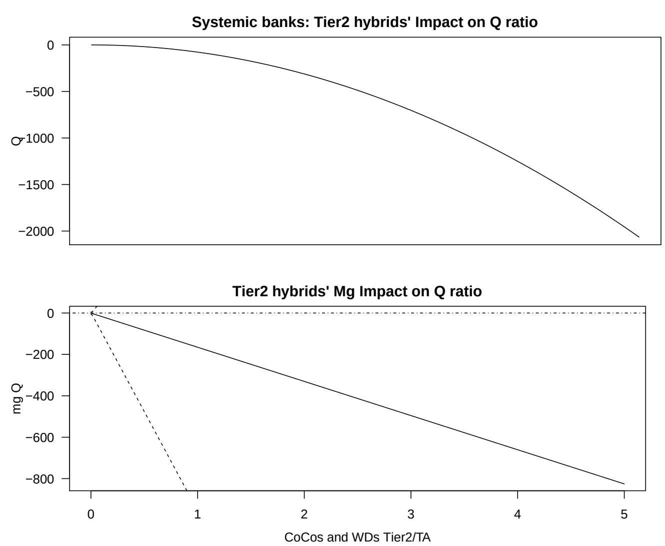


Figure 32: Systemic banks: Tier2 hybrid instruments' impact over Q ratio.

19.1 Robustness check: the model Setup and results

19.1.1 Bank Structure

The Bank Structure is considered in a continuous time framework, with initial date $t = 0$ and terminal date $t = T$. We focus on $Fr(T)$, i.e. future growth opportunities. They materialize only at the end of the period, T, but the franchise value might vanish previously, as soon as the liabilities exceeds the asset value in $0 \leq t \leq T$, that is when

$$\tau_{Fr=0} = inf\left\{t \geq 0 : A(t) \leq MV^{SD} + Dep\right\}. \tag{44}$$

We call the MVA the sum of the tangible value of the Assets and franchise value. Their dynamic is:

$$d\ln(MVA(t)) = \left(\mu_{MVA_t} - \frac{\sigma^2_{MVA}}{2}t\right)dt + \sigma_{MVA}dB_t, \tag{45}$$

where B_t is a standard Brownian motion, the drift, μ_t, is time-varying and σ is constant and both are referred to the sum of the tangible value of the assets and the franchise value. Similar to Babbel and Merrill (2005) and Barone-Adesi et al. (2014), we split the value of the bank into three components. First, considering the limited liability, the market value of the equity of our bank is a **call option** on the assets: $E(T) := max(A(T) - L)$, where A is the value of the banks' assets and L the face value of the liabilities.Second, let's split the value of equity into the following two components: $E(T) := X(T) + Put^{def}(T)$**, where** $X(T) := A(T) - L$ **is the net tangible value** of the bank, without considering the limited liability, which is represented through the **default put option**. Third, we allow the bank to be able to invest in value creating opportunities at time T, through the introduction of the **franchise value** $Fr(T)$. Hence, $E(T) := X(T) + Put^{def}(T) + Fr(T)$. Taking into account the different sources of financing for our bank (deposits, standard debt and CoCos), the end of the period equity market value is given by the three components: the net tangible value, the shareholders' option to default, the franchise value.

$$\begin{aligned} Tier1(T) = A(T) - Dep - FV^{SD}(T) \\ +Fr(T) + Put^{def}(T). \end{aligned}$$

CoCos conversion occurs at

$$\tau_{Conv} = inf\{t \geq 0 : A(t) \leq V_{Conv}\}.$$

19.1.2 Pricing the default option

Potential arbitrage opportunities, that could arise buying the bank and selling short the tangible assets and the franchise value. To prevent arbitrage: the underlying is given by the sum of both franchise value and market value of the assets. The extended standard pricing:

$$\begin{aligned} Put^{def}(lev, \sigma_{MVA}, rf) &= \left(MV^{SD} + Dep\right)\Phi(-d_2) + \\ &\quad (-(MVA)\Phi(-d_1)), \\ &\quad \text{with } \{\tau_{Fr=0} > T\}, \\ where \quad & d_1 = \left(\frac{ln\left(\frac{1}{1-lev}\right) + \left(rf + \frac{\sigma^2_{MVA}}{2}\right)T}{\sigma_{MVA}\sqrt{T}}\right), \\ lev = \left(\frac{Tier1}{MVA}\right), \quad & d_2 = d_1 - \sigma_{MVA}\sqrt{T}, \Phi - standardNormal \end{aligned} \tag{46}$$

The greeks for this option are given as follows:

$$\begin{aligned} Sensitivity \quad to \quad leverage \quad &: \left[\frac{\delta Put^{def}_{i,t}}{\delta lev_{i,t}}\right] < 0 \\ Sensitivity \quad to \quad volatility \quad &: \left[\frac{\delta Put^{def}_{i,t}}{\delta \sigma_{MVA_{i,t}}}\right] > 0 \\ Sensitivity \quad to \quad policy - rate \quad &: \left[\frac{\delta Put^{def}_{i,t}}{\delta rf_{i,t}}\right] < 0 \end{aligned} \tag{47}$$

19.1.3 Pricing the DOC option, in presence of non-observable underlying

The unobservable value of potential growth is Fr, which is net of investment costs, thus the strike is set to zero. When the bank does not default, $MVA(0) > MV^{SD}$, the pricing is:

$$DOC\left(lev,\sigma_{MVA},rf\right) \qquad = Fr\left[\Phi\left(v_1\right)+ - \left(1-lev\right)^{2\lambda}\Phi\left(y_1\right)\right]$$

$$with\left\{\tau_{Fr=0}>T\right\}, \tag{48}$$

$$where \qquad \lambda=\frac{rf+\frac{\sigma_{MVA}^2}{2}}{\sigma_{MVA}^2}$$

$$v_1=\frac{ln\left(\frac{1}{1-lev}\right)}{\sigma_{MVA}\sqrt{T}}+\lambda\sigma_{MVA}\sqrt{T}, \quad y_1=\frac{ln(1-lev)}{\sigma_{MVA}\sqrt{T}}+\lambda\sigma_{MVA}\sqrt{T}$$

The standard greeks for this option are given as follows:

$$\begin{aligned} &Sensitivity \quad to \quad leverage &:& \left[\frac{\delta DOC_{i,t}}{\delta lev_{i,t}}\right]>0 \\ &Sensitivity \quad to \quad volatility &:& \left[\frac{\delta DOC_{i,t}}{\delta\sigma_{MVA_{i,t}}}\right]>0 \\ &Sensitivity \quad to \quad policy-rate &:& \left[\frac{\delta DOC_{i,t}}{\delta rf_{i,t}}\right]>0 \end{aligned} \tag{49}$$

19.1.4 The optimization problem

We split the optimization problem for risk appetite into two steps.
The first step.
We estimate the unobservable franchise value and the market value of the assets, which are embedded in the equity market value. We minimize the distance between the data concerning the MVE and the model, through the non linear least squares criterion function. We perform a step by step optimization for $\Theta_{i,t} := Fr_{i,t}, A_{i,t}, \sigma_{MVA_{i,t}}$, solving simultaneously:

$$\begin{cases} e_{1,i,t}= & MVE_{i,t}-(A_{i,t}-MV(D+L)+DOC+Put^{def}), \\ e_{2,i,t}= & \sigma_{MVE_{i,t}}MVE_{i,t}-\sigma_{MVA_{i,t}}(MVA_{i,t})\Phi(d_{1i,t}), \end{cases} \tag{50}$$

The non linear least square function is the following:

$$\Theta_{i,t}^{*}=\underset{(\Theta_{i,t})}{arg\min} \quad \sum_{j,i,t=1}^{2,n,m}\left[e_{j,i,t}^2\right] \tag{51}$$

The second step.

We look for the optimal level of leverage, assets' and franchise value's volatility and policy rate

$(\Theta_{i,t} := \left(lev_{i,t}, \sigma_{MVA_{i,t}}, rf_{i,t}\right))$ that simultaneously optimize the objective function $(O.f.)$, defined as:

$$O.f._{i,t} := \frac{DOC\left(\Theta_{i,t}\right) + PUT^{def}\left(\Theta_{i,t}\right)}{A_{i,t}}. \tag{52}$$

When $MV^{SD} < MVA\left(0\right)$, the optimization problem is:

$$\Theta^*_{i,t} = \underset{\Theta_{i,t}}{arg\max} \quad \left[Of_{i,t}\right] \tag{53}$$

The shape of risk appetite is assessed through the determinant of the hessian matrix in a three-dimensional perspective. Setting the sensitivity to leverage, vega and rho equal to zero.

$$\begin{aligned} &leverage - driven \quad R.A.: \left[\frac{\delta Of_{i,t}}{\delta lev_{i,t}}\right] = 0 \mid_{\sigma^*_{MVA_{i,t}}, rf^*_{i,t}} \\ &volatility - driven \quad R.A.: \left[\frac{\delta Of_{i,t}}{\delta \sigma_{MVA_{i,t}}}\right] = 0 \mid_{lev^*_{i,t}, rf^*_{i,t}} \\ &policy - rate - driven \quad R.A.: \left[\frac{\delta Of_{i,t}}{\delta rf_{i,t}}\right] = 0 \mid_{lev^*_{i,t}, \sigma^*_{MVA_{i,t}}} \end{aligned} \tag{54}$$

Estimating the optimal value of CoCos to issue:

$$CoCo_{i,t} := \left(lev^*_{i,t} - lev^{act}_{i,t}\right), \tag{55}$$

19.1.5 The results: a comparison

Table 38: Summary statistics of the key optimized variables

Statistic	N	Mean	St. Dev.	Min	Median	Max
Opt_Vol	1135	0.036	0.015	0.0001	0.025	0.076
Opt_lev	1135	0.077	0.048	0.0002	0.080	0.167
Opt_ADTier1	1135	0.017	0.056	−0.136	0.020	0.134

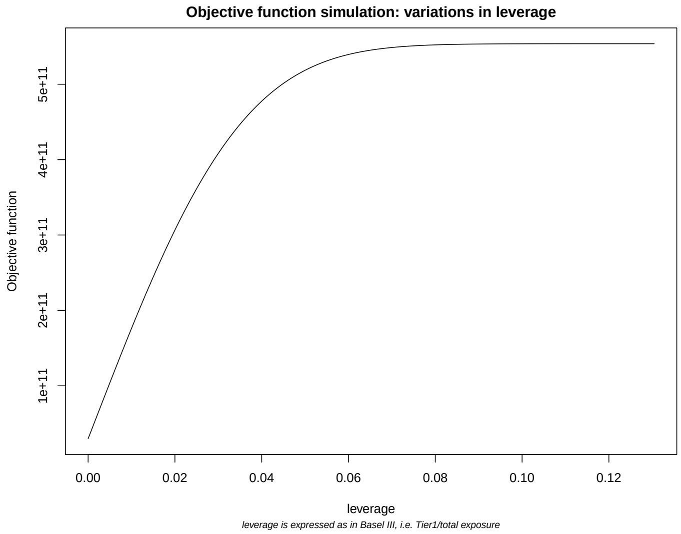


Figure 33: Objective function optimization, maximum reached for an average leverage of 7% to 8%, that is given our results an average issue of CoCos and WDs belonging to AD Tier 1 of 1.7% and a median optimal volatility of 2.5%.

Conclusions

In summer 2017, after the high school final exam, I was at the seaside discussing with my mother about my future. She asked why I was interested in studying economics. I answered that this was the *medium* I preferred to understand the world where I was living. At that time, newspapers, and all over the media, you could find many articles and pieces of news about worldwide economics and finance. It was the beginning of the crisis. I had the privilege to hear about the failure of Lehman Brothers during microeconomics class, early in my second year of the Bachelor. I finished one term in advance my Master in Banking and Finance, a couple of days before my 23rd birthday. I was very curious and motivated to study and had the privilege to enter the SFI PhD Student Program at USI. Learning from the crisis makes everyone better off. Overall, we show that Basel III rule proposes a great incentive pushing the decrease of the RWAs and the increase in solidity, via the promotion of the enlargement of the Tier 1. However, there is still room for pushing banks to decrease the riskiness of its assets. Theoretically, we find that hybrids are helpful and promote the increase in bank value itself only if associated with a relative low level of volatility. Empirically, we find that, on one side, the relation between hybrids and bank riskiness is described by a U-shaped curve and, on the other side, with respect to growth opportunities, we have a inverted-U shaped curve, leading to a trade-off. Hence, banks should make an effort in finding a balance between containing bank riskiness and contemporaneously promoting bank growth. We leave unanswered questions and, above all, it should be interesting to assess more deeply the role of the monetary policy, accounting for other relevant variables, such inflation rate or unemployment rate.

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