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Navigating Peaks and Troughs: Can We Leverage Volatility Metrics in Passive Dynamic Asset Allocation ?

Bachelor's Thesis completed in partial fulfillment of the requirements for the Bachelor HES

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Declaration

This Bachelor thesis is submitted as part of the final examination at the Haute Ecole de Gestion Geneva (HEG), in pursuit of the Bachelor of Science degree in Business Administration.

I certify that I have completed this work independently, without using any sources other than those cited in the bibliography. Furthermore, I attest that the submitted work is the result of my personal reflection and has been written autonomously. This work has also been analyzed using the plagiarism detection software recommended by the program.

The use of the conclusions and recommendations formulated in the Bachelor thesis, without prejudice to their value, does not engage the responsibility of myself, the thesis advisor, the juror, or the Haute Ecole de Gestion Geneva (HEG).

Coding & AI Usage Declaration

I hereby declare that in the course of developing this Bachelor thesis, I have utilized only one AI tool, ChatGPT, for strengthening my analyses in R coding and for corrections and error message solving in my R codes. I have not used any AI, including ChatGPT, to source information for this thesis.

This work remains my own, reflecting my personal research and efforts, with AI assistance limited to the specific purposes mentioned above.

The coding specifics have been executed with the kind help of Dr. François Duc with application of the principles taught during his course titled "*Pratique de la Gestion du Risque sur R*" at Haute Ecole de Gestion Geneva (HEG).

The details of the R codes are available upon request to Dr. François Duc (email address: francois.duc@hesge.ch).

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Executive Summary

This thesis explores Volatility-Based Dynamic Asset Allocation (DAA) strategies with the goal of analyzing various volatility metrics, evaluating their predictive power, and developing models to enhance portfolio performance and risk management.

Part I analyzes current S&P500 market dynamics. This section provides an analysis of various volatility metrics and of the CBOE VIX Index (VIX). **Part II** conducts an empirical study on the predictability of the VIX, by comparing the VIX with various calculated volatilities (historical volatilities and GARCH models). The Variance Premium (VP), which predicts future volatility, is calculated by subtracting rolling historical volatility from implied volatility. **Part III** develops volatility-based DAA models using R. An ETF portfolio is constructed with portfolio re-allocation rules defined for different volatility regimes and metrics. Each end-strategy is composed of one allocation set and one volatility metric.

- **Sets of allocations:** ***MONTBLANC*** (initial allocations) and ***ZERMATT*** (designed to improve risk metrics).
- **Strategies:** ***BASIC*** (re-allocation based on VIX levels), ***PEAK*** (re-allocation based on 252-day VP levels) and ***EDGE*** (re-allocation based on 21-day VP levels).

<u>Key Findings:</u>

VIX Predictability: The results show that the VIX's predictive power is limited due to its strong correlation with short-term historical volatility. Comparing the VIX to GARCH models proved that complex models offered little improvement. When the VP is calculated using short-term volatility – given that the VIX closely tracks it – it evaluates the predictive value in the small difference between the VIX and the short-term historical volatility, essentially becoming the difference between short and long-term historical volatility.

Backtest results: PEAK provides the best Sharpe Ratios, often outperforming all benchmarks. **BASIC** offers a balanced approach but with moderate results. **EDGE** showed poor returns with high volatility. Considering end-strategies, **MONTBLANC PEAK** shows the highest results, outperforming all benchmarks out-sample. We also optimized allocation sets and volatility thresholds, both showing robust out-sample results, which is uncommon as optimizations often underperform on future data.

Conclusion: This study emphasizes the importance of integrating various volatility metrics to develop more resilient portfolios. Future research could explore more sophisticated models and include behavioral finance aspects and active decision-making to build more effective and adaptative portfolio management strategies.

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1. PART I – SETTING THE SCENE

1.1 Introduction

As we enter the second half of 2024, financial markets are marked by both optimism and caution. The U.S. economy is more stable as the labor market has returned to 2019 levels and core PCE[1] inflation has moderated to 2.6% from a peak of 5.6%. The Federal Reserve is expected to begin cutting rates carefully, likely starting in September. Corporate earnings were strong in Q1, particularly from the Magnificent Seven[2] (Pease 2024).

While a recession seems unlikely this year, macroeconomic uncertainty remains high. Global growth is slowing down from its strong pace in 2023 but is becoming more stable. The world is becoming less dependent on U.S. demand as European consumers and global capEx[3] drive a modest manufacturing recovery. Even though early-year inflation is fading, core inflation is expected to stay around 3% globally due to tight labor markets and wage gains, limiting the room for policy easing (Pease 2024).

Equity valuations are high, and credit spreads are tight, with market psychology showing optimism without extreme euphoria. Treasury yields are volatile, but bonds are seen as a good medium-term value. The "*higher for longer*" rate environment has favored large, quality companies, particularly in the tech sector. However, the concentration of gains in a few mega-cap stocks raises sustainability concerns (Dr. Kelly 2024).

Additionally, regional and political developments are influencing markets. European Parliament elections maintained the center-right status quo but signal potential deeper changes in France and Germany (Dr. Hechler-Fayd'herbe 2024).

In this financial landscape, major themes like demographic changes, decarbonization, and technological advancements, especially AI, present new opportunities and challenges, that require a dynamic and informed investment strategy. The volatility and unpredictability of the markets highlight the limitations of traditional passive investment strategies. A passive, volatility-focused approach to dynamic asset allocation offers a promising method for managing market fluctuations, providing a cost-effective and flexible framework for improved risk management and investment performance.

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[1] Core CPE is the Personal Consumption Expenditures Price Index. This index excludes food and energy.
[2] Comprises Alphabet, Amazon, Apple, Meta Platforms, Microsoft, NVIDIA and Tesla.
[3] Capital expenditures (CapEx) are funds a company uses to acquire, upgrade, and maintain physical assets like property, buildings, technology, and equipment. They support new projects or investments.

1.1.1 Hypothesis

In this period characterized by the financial markets' unpredictable fluctuations, this bachelor thesis argues for the adoption of a passive Volatility-Based Dynamic Asset Allocation (DAA) strategy, with a particular focus on utilizing the CBOE Volatility Index (VIX) as the primary factor and defined Variance Premiums as secondary factors (detailed in the thesis). These strategies are proposed as an innovative approach to improve portfolio management with the aim to harness the potential for higher returns while simultaneously reducing operational costs and strengthening risk management practices. The central thesis is built on the idea that the VIX, as an indicator of the market's expected volatility (CBOE Not dated), can act as a key tool for modifying asset allocations in a way that is adaptive.

1.1.2 Objectives

Objective 1: Comprehensive Analysis of the VIX and Volatility

The first objective is to thoroughly examine the VIX, including its relationship with the S&P 500, on which it is based. This includes a deeper understanding the S&P 500, with its current composition and current state. It also includes the understanding general volatility concepts, different types of volatility, and how the VIX is constructed. We will also explore how the VIX relates to different asset classes. This analysis aims to show how changes in the VIX can signal adjustments in a portfolio to improve investment outcomes.

Objective 2: Assessing the Predictability of the VIX

The second objective is to evaluate how well the VIX predicts future market volatility compared to other models like GARCH and rolling volatilities. This involves comparing the VIX's forecasting ability to these models to see if it provides better predictions for market movements. This second objective also includes the definition and calculation of Variance Premiums (anticipatory element of implied volatility).

Objective 3: Building and Testing Volatility-Based Dynamic Asset Allocation Models

The third objective is to create simple models using VIX-based and Variance-Premium-based dynamic asset allocation (DAA). We will backtest various strategies using historical data to see how these perform, and we will also create optimizations. These analyses aim to demonstrate the effectiveness of using the VIX and Variance Premiums for dynamic portfolio management.

1.2 Dynamic Asset Allocation

Dynamic Asset Allocation (DAA) is a tactical investment strategy that adapts to fluctuating market conditions with the aim of either boosting returns or reducing risk over a timeframe. This approach contrasts with traditional static asset allocation strategies that maintain a consistent distribution across different asset classes regardless of changing market dynamics. A DAA usually modifies asset allocations based on new market predictions, economic developments, or valuation indicators (Chen 2022a).

1.3 Portfolio Management – Active vs. Passive Strategies

Active and passive fund management represent distinct strategies for investing, each having its advantages and disadvantages. Active investing involves a hands-on approach where portfolio managers actively select stocks or other assets with the aim of outperforming the market average. This method necessitates in-depth analyses and the expertise to make timely decisions. While active management allows for flexibility and targeted risk management, it is generally more costly due to higher transaction fees and the expenses associated with employing a team of research analysts (Investopedia 2023).

On the other hand, passive investing focuses on long-term holdings with minimal trading, often through index funds that mirror major market indices like the S&P 500. This strategy benefits from lower management fees, and transparency, as the investments align with a predetermined benchmark. However, passive investing limits investors to the returns of the market indices they track, with no potential to outperform the market.

Despite the potential for higher returns from active management, empirical evidence suggests that passive investments have historically performed better and attracted more investment flows. This trend can be attributed to the consistent underperformance of active managers compared to their benchmarks over the long term. Over the past 15 years, more than 87% of US Large-Cap funds have underperformed the S&P 500 (SP Global Website 2023). Nonetheless, some investors prefer active management to achieve specific investment goals, such as hedging strategies (Investopedia 2023).

1.4 Analysis of the S&P 500 Index

Before exploring how a DAA strategy can be built around the VIX, it is crucial to understand the fundamentals and complexities of the S&P 500 and the VIX themselves.

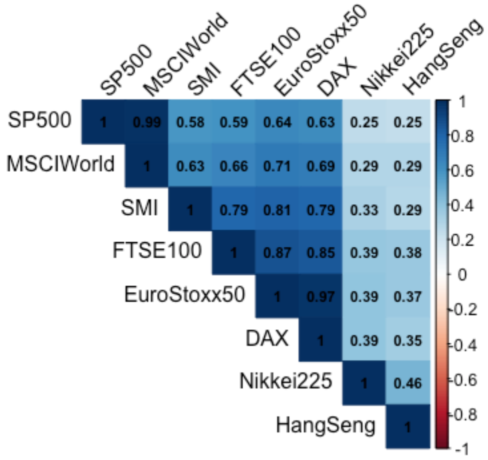
The S&P 500 is widely known as a barometer for the overall U.S. economy, frequently used by investors to benchmark the performance of their portfolios. Its status over other stock indices arise from its comprehensive scope, making it a preferred choice for investors, analysts, and fund managers to evaluate the economic climate and measure their performance against it, aiming to achieve returns that surpass the index.

One of the main benefits of using the S&P 500 as a benchmark is its inclusion of a broad array of large-cap companies, offering a panoramic view of the U.S. economic health across diverse sectors. This broad market exposure is further enhanced by the index's methodology of updating its components quarterly.

The election for the index is determined by a committee that considers criteria such as a minimum market capitalization of $14.5 billion, a public float[4] of at least 50%, and U.S. headquarters. Additionally, companies are required to have a trading history of at least 12 months post-IPO before being eligible for inclusion in the index. This process ensures that the S&P 500 remains a current and comprehensive reflection of the U.S. large-cap market (SPGlobal, Methodology 2024).

However, the S&P 500 might not fully represent the investment landscape for many investors who diversify across assets beyond stocks, including bonds, precious metals, and cash. Moreover, portfolios may contain small-cap or international stocks not covered by the index.

Figure 1 - Correlation Matrix : S&P500 vs. Major Indices (From 01.01.2020 to 05.07.2024)



The above analysis provides a current state of the correlation between the S&P 500 and major indices. The dates are not within the in/out-sample periods (defined in following sections) as this correlation matrix only provides a deeper understanding of the S&P 500 and does not bias our models and strategies.

The correlation analysis between the S&P 500 and other major global indices highlights how interconnected the global financial markets are today. The high correlation between

[4] Public float is the portion of a company's shares available for public trading.

the S&P 500 and the MSCI World Index (0.99) indicates that the performance of U.S. large-cap stocks is a significant driver of global equity performance. This is because the MSCI World Index includes a large proportion of U.S. companies, reflecting the dominance of the U.S. economy in the global market.

European indices, such as the Euro Stoxx 50, FTSE 100, and DAX, show moderate to high correlations with the S&P 500 (around 0.59 to 0.64). This indicates that while these markets are influenced by their regional economic conditions, they also move in tandem with the U.S. market to a significant extent. The high correlation can be explained by the interconnectedness of global financial markets and the synchronized nature of economic cycles rather than a direct influence of the U.S. market.

Asian markets, represented by the Nikkei 225 and Hang Seng, exhibit lower correlations with the S&P 500 (0.25). This lower correlation highlights the potential for diversification benefits when including Asian equities in a portfolio that is heavily weighted towards U.S. stocks. The distinct economic environments, regulatory landscapes, and market dynamics in Asia contribute to these differences.

1.4.1 Composition and Cycles

The S&P 500 is a market capitalization-weighted index, meaning that companies with larger market capitalizations have more influence on the index's performance.

As of the latest data, the sector breakdown of the S&P 500 is as follows (CFI Team Not dated):

- **Information Technology:** This sector holds the largest weight in the index, driven by tech giants such as Apple and Microsoft.
- **Health Care:** Companies like Johnson & Johnson, Pfizer, and Merck are key contributors to this sector.
- **Financials:** Major financial institutions like JPMorgan Chase, Bank of America, and Wells Fargo dominate this sector.
- **Consumer Discretionary:** This includes companies such as Amazon, Home Depot, and McDonald's.
- **Communication Services:** Facebook (Meta), Alphabet (Google), and Netflix are significant players in this sector.
- Other sectors include Industrials, Consumer Staples, Utilities, Real Estate, Materials, and Energy.

Figure 2 - Top 10 S&P500 Sectors, Weights and Type (as of July 1, 2024)

Top 10 S&P 500 Sectors, Weights, and Types

Sector	Weight (%)	Type
Information Technology	33.9	Cyclical
Financials	11.8	Cyclical
Health Care	11.7	Defensive
Consumer Discretionary	10.0	Cyclical
Communication Services	9.2	Defensive
Industrials	7.6	Cyclical
Consumer Staples	5.8	Defensive
Energy	3.5	Cyclical
Utilities	2.4	Defensive
Real Estate	2.1	Defensive

The above figure provides a current state of the S&P 500. The date is not within the in/out-sample periods (defined in following sections) as this table only provides a deeper understanding of the S&P 500 and do not bias our models and strategies.

Figure 2 shows the Top 10 S&P 500 sectors analysis with weight (Tun 2024) and typical business cycle , as described below:

- **Cyclical :** Cyclical sectors are those whose performance is highly correlated with the economic cycle. These sectors tend to perform well during periods of economic expansion and underperform during economic contractions (MSCI 2023).
- **Defensive :** Defensive sectors, on the other hand, tend to have more stable performance regardless of the economic cycle. These sectors perform relatively well during economic downturns and provide a buffer against economic volatility (MSCI 2023).

1.4.2 Key Metrics Analysis

- **P/E Ratio (24.79 as of December 2023):** Indicates high investor expectations for future growth. However, it may suggest overvaluation if growth expectations are not met (Fernando 2024a).
- **Dividend Yield (1.35% as of March 2024):** Relatively low, suggesting companies are reinvesting earnings rather than returning them as dividends.

- **P/B Ratio (4.312 as of December 2023):** The price-to-book (P/B) ratio compares a company's market value to its book value by dividing the stock price per share by the book value per share. It is used to identify undervalued stocks, when their P/B ratio is below 1.0 (Fernando 2024b).

- **Market Capitalization (44.08 trillion USD as of March 2024):** Highlights the significant size and influence of the S&P 500 in the global market.

The above metrics provide a current state of the S&P 500. The dates are not within the in/out-sample periods (defined in following sections) as these metrics only provide a deeper understanding of the S&P 500 and do not bias our models and strategies.

These metrics indicate a market with high growth expectations and strong profitability but also potential overvaluation concerns.

1.4.3 Historical Performance of the S&P 500 During the In-Sample Period[5]

Between January 2004 and December 2018, the S&P 500 has experienced significant volatility and major financial crisis. This period starting just after the early 2000s recession witnessed a steady growth until the 2008 Great Recession. This crisis, triggered by the collapse of the company Lehman Brothers and a crisis within the global banking system, led to a deep decline of the S&P 500. It lost approximately 57% of its value from its peak in October 2007 to its valley in March 2009. This period was characterized by a very high volatility on the markets and was a pivotal test of investor's resilience.

After the financial crisis, the S&P 500 entered a prolonged bull market, thanks to significant monetary support from major central banks like the Federal Reserve, which introduced quantitative easing programs. In 2011, the European sovereign debt crisis briefly shook investor confidence, causing temporary declines in the S&P 500 amid fears of a potential breakup of the Eurozone. However, the U.S. market quickly recovered, driven by strong corporate earnings and improving economic conditions.

Throughout the remainder of the in-sample period, the S&P 500 experienced mostly consistent growth, with occasional minor setbacks. Notably declines occurred in 2015–2016, prompted by concerns over global economic slowdowns and declining oil prices. Despite these challenges, the index maintained an overall upward trend, concluding the period in 2018 with positive momentum, supported by robust economic indicators and

[5] In-Sample Period refers to period starting ***January 2004 to end of December 2018***. This In-Sample Period has been used in Part II of this thesis for all descriptive statistical analyses and estimations.

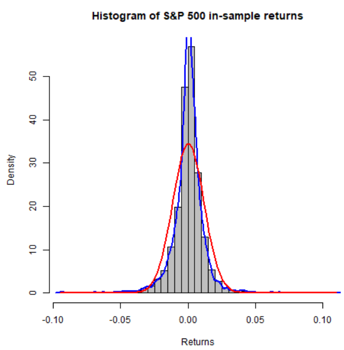
optimism surrounding fiscal policies. This period of recovery and expansion underscores the resilience of the market in the face of various challenges and periods of uncertainty.

Figure 3 - S&P 500 Evolution (In-Sample)

1.4.4 Univariate Descriptive Statistics of the S&P 500 during the In-sample Period

During the in-sample period, the S&P 500 recorded an annualized return of 5.33% and an annualized volatility of 18.36%. Considering the average annual risk-free rate from one-month T-bills for the sample, the Sharpe ratio stands at 0.23. Consistent with extensive literature, the distribution of S&P 500 returns is not normal, exhibiting heavy tails (Excess Kurtosis = 8.48) and a slight negative skewness (Skewness = -0.3). The Jarque-Bera[6] test rejects the normality of the distribution[7].

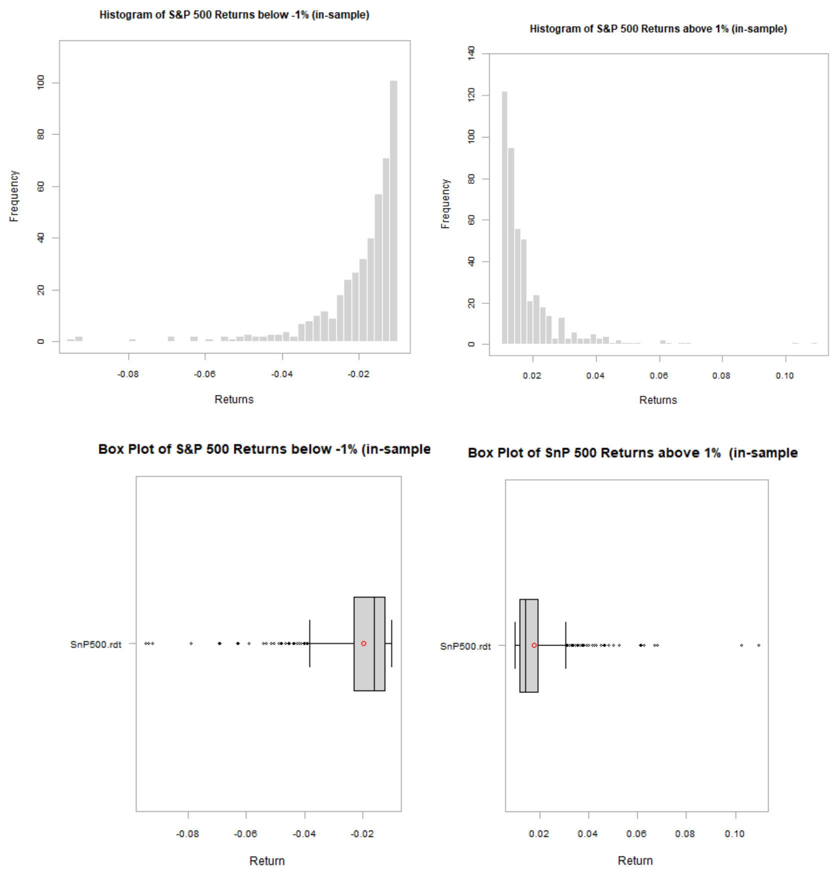
Figure 4 - S&P 500 Distribution (In-Sample)



[6] The Jarque-Bera test is calculated using these skewness and kurtosis values, following a chi-squared distribution under the null hypothesis. For a normal distribution, the skewness should be close to zero and the kurtosis should be close to 3.
[7] See table A01 and A02 in Appendix 1.

Around 12% of the days experienced returns below -1% (with an average return of -2%), and nearly 12% had returns above 1% (with an average return of about 1.8%). These figures illustrate the excess kurtosis in the data. It underscores the importance of protecting against extreme losses while capitalizing on significant gains.

Figure 5 - S&P 500 Tail Distributions (In-Sample)



1.5 Analysis of the Volatility

Volatility analysis delves into the price changes of securities over time, evaluating the magnitude and rapidity of these fluctuations. Characterized by large price movements, high volatility signals a less predictable price trend, whereas low volatility is indicative of a more stable pricing path. This type of analysis is grounded in the examination of historical price data, with statistical methods like standard deviation being employed to quantify volatility. A significant divergence in prices from their average suggests elevated volatility, which may imply higher risk but also the potential for increased returns. On the other hand, minimal volatility points to lower risk and steadier returns. Traders utilize volatility analysis to identify potential trading opportunities and to create strategies for risk management (Remesh, Gaba 2024).

Various approaches to volatility analysis exist, including historical volatility, which looks at past price changes and implied volatility, which uses options pricing to forecast future volatility. Influential factors on stock volatility include economic updates, earnings announcements, sector-specific developments, interest rate shifts, political events, and general market sentiment, with small caps typically experiencing greater volatility than large caps (Remesh, Gaba 2024).

The standard deviation stands out as the primary tool in volatility analysis, reflecting the degree of price variation from the mean. Other crucial metrics involve the variance, which measures the spread between numbers in a data set, the beta, which evaluates a stock's relative volatility to the market and the Value at Risk (VaR), which calculates the maximum potential loss of a portfolio (Remesh, Gaba 2024).

Major factors influencing stock market volatility (Remesh, Gaba 2024):

i. **Economic Updates:** Economic indicators like job reports, GDP figures, inflation rates, and consumer sentiment surveys strongly influence market movements. When these indicators differ significantly from expectations, they can cause market shifts. Positive economic news boosts investor confidence, reducing market volatility. Conversely, weaker reports increase volatility and uncertainty.

ii. **Federal Reserve Actions:** The decisions made by the Federal Reserve regarding interest rates and monetary policies directly influence stock valuations, risk appetite, and market volatility. Policies that accommodate economic growth, such as rate cuts, often lead to decreased volatility, whereas tightening measures like rate hikes generally provoke increased market volatility.

iii. **Global Political Landscape:** Major geopolitical events such as elections or conflicts introduce uncertainty and volatility into markets. Investors typically react negatively to instability and changes in policies. Instances where geopolitical tensions ease tend to stabilize markets, whereas conflicts and uncertainty trigger spikes in volatility.

iv. **Corporate Performance:** Regular updates on the quarterly earnings of major corporations offer insights into their financial health and growth prospects. Positive earnings reports and optimistic guidance tend to reduce volatility by indicating robust business conditions. Conversely, disappointing results or cautious forecasts can increase volatility by raising uncertainty among investors.

v. **Investor Sentiment:** Optimistic investor sentiment stabilizes markets, reducing volatility as confidence strengthens. Negative sentiment increases volatility, as it fosters fear and uncertainty. Significant shifts in sentiment typically precede changes in volatility, as markets seek balance.

vi. **Financial Turmoil:** Severe events like recessions, market crashes, or credit crises sharply raise volatility due to increased uncertainty and widespread selling as investors sell off assets.

Most common statistical models used in volatility analysis (Remesh, Gaba 2024):

i. **Simple Moving Averages[8]:** When examining a security's price over a specific timeframe, the moving average serves as a fundamental indicator of volatility. Shorter periods focus on immediate volatility, while longer periods capture trends in long-term volatility.

ii. **Weighted Moving Averages:** By assigning greater weight to recent data and less to older data, you can respond more quickly to changes in volatility. Exponentially weighted moving averages achieve this efficiently.

iii. **Bollinger Bands:** These bands, created by applying moving averages several standard deviations above and below a price plot, highlight periods of high and low volatility. Wider bands indicate higher volatility.

iv. **ARCH/GARCH Models:** These models forecast volatility based on past patterns of volatility clustering and mean reversion. *Generalized Autoregressive Conditional Heteroskedasticity* incorporates external factors like interest rates.

v. **Value at Risk (VaR):** This metric estimates volatility-adjusted maximum loss thresholds for a position or portfolio at a given confidence level, aiding in the quantification of downside risk.

1.5.1 Historical Volatility

Historical volatility quantifies how much a security's price has varied over a set time, showing how fast and unpredictable its past movements were. It is calculated by finding the standard deviation of the security's price changes over a specified period, commonly 30, 60, or 90 days.

[8] Simple Moving Averages (SMA) smooth out price data by calculating the average price of a security over a specific number of periods, helping to identify trends.

This calculation, typically annualized to project the volatility over a year, employs the following formula:

Equation 1 - Annualized Historical Volatility

Ann. Hist. Vol. = Standard Deviation of Returns × √(Number of Periods per Year)

For example, calculating a stock's 30-days historical volatility involves determining the standard deviation of its daily returns over 30 days and then annualizing this figure by multiplying by the square root of 252, the approximate number of trading days in a year.

This method provides a clear way to measure and compare how much different securities or indices have fluctuated. Higher historical volatility suggests larger price swings and more variability, while lower volatility indicates steadier and more predictable price patterns (Chen 2022b).

Historical volatility's key strength lies in its reliance on actual past price movements, offering a concrete measure of past volatility without theoretical assumptions. However, its main drawback is its backward-looking nature, which may overlook recent or upcoming shifts in market conditions.

In contrast, implied and future-realized volatilities take a forward-looking approach. Implied volatility reflects expectations of future market fluctuations derived from derivative pricing. Future-realized volatility, using models like GARCH, predicts future volatility by analyzing historical patterns. These methods integrate market sentiment, assumptions, and predictive analytics, unlike historical volatility, which focuses solely on analyzing past events (Chen 2022b).

1.5.2 Implied Volatility

Implied volatility predicts future stock volatility based on option prices, reflecting market expectations. It contrasts with historical volatility, which looks at past stock price changes.

To find implied volatility, the Black-Scholes model is often used. This model calculates option values using inputs like current stock price, strike price, time until expiration, interest rates, and an estimated volatility. Implied volatility is derived by entering the option's market price into the model along with these inputs. The volatility value that makes the model's price match the market price of the option is considered the implied volatility (Ganti 2024).

A higher implied volatility suggests that the market anticipates greater fluctuations in the stock price, whereas a lower implied volatility points to expectations of less dramatic changes. Implied volatility tends to increase in times of market uncertainty and decrease when the market is calm (Ganti 2024).

Implied volatility is crucial for traders for several reasons:

- **Options Trading:** It affects option pricing, helping traders decide when to buy or sell options. Higher implied volatility can lead to more expensive options, suggesting a potential strategy of selling options, while lower implied volatility might make buying options more attractive.

- **Portfolio Hedging:** By tracking implied volatility, traders can judge whether options are relatively overpriced or underpriced, making hedging strategies against stock price movements more effective.

- **Volatility Forecasting:** Implied volatility provides insights into expected future volatility, reflecting the market's collective sentiment, and offering a forward-looking perspective, unlike historical volatility that is anchored in past price movements.

- **Sentiment Analysis:** The level of implied volatility can signal the mood among options traders, with high levels indicating fear or uncertainty, and low levels suggesting increased confidence.

The key difference between implied and historical volatility is their orientation in time: implied volatility projects forward, indicating what the market expects will happen, while historical volatility looks backward, analyzing what has already occurred. While historical volatility quantifies volatility based on actual past price data, implied volatility, calculated from options prices, offers a prediction of how volatile the market expects the stock to be in the future (Fidelity HK Not dated).

1.5.3 Future-Realized Volatility

Future-realized volatility, or future volatility, is the actual volatility observed over a future period. It is important for comparing with implied volatility to assess how accurately the options market predicts upcoming volatility (Remesh, Gaba 2024).

Future-realized volatility is essentially historical volatility calculated for a future period after it has occurred. For instance, to find the future-realized volatility of a stock over the next 30 days, one must wait until this period ends to analyze the stock's price changes using a formula like standard deviation of returns or variance. This figure shows the actual volatility experienced by the stock during that future timeframe (Jacobson 2020).

Comparing future-realized volatility with implied volatility is crucial for evaluating options pricing models. Implied volatility forecasts future volatility based on current option prices, while future-realized volatility shows the actual volatility experienced. Analyzing the relationship between these two figures helps assess how accurately market expectations align with actual volatility (Jacobson 2020).

In summary, implied volatility reflects market expectations for future volatility, while future-realized volatility shows the actual volatility observed over a set period. Comparing these measures helps investors and traders evaluate how well implied volatility predicts actual market behavior. This assessment guides more informed decisions in trading and risk management strategies.

1.6 The CBOE Volatility Index (the "VIX")

The VIX is a real-time index that represents the market's expectations for the relative strength of near-term price changes of S&P 500. Because it is derived from the prices of S&P 500 options with near-term expirations dates, it generates a 30-day forward looking projection of volatility. The VIX attempts to measure the magnitude of price movements of the S&P 500. The more dramatic the swings, the higher the level of volatility, and vice versa. In addition to being an index to measure volatility, investors can also trade VIX futures, options and ETFs to hedge or speculate on volatility changes in the index (CBOE Not dated).

1.6.1 Why the VIX and not Another Volatility Index ?

Choosing the VIX over other indexes for adjusting investments is a prudent decision for several reasons. First, the VIX, often called the "fear gauge," predicts how much the stock market might swing soon by looking at options for the S&P 500. It is a forward-looking tool that captures the mood of investors, showing whether they are feeling confident or scared. This makes it an essential indicator for fund managers who need to quickly adjust their strategies based on market sentiment (Sun Life Global Investments 2023).

The VIX updates in real-time, which is great for making fast, informed decisions about where to move money. Its calculation (detailed in the following section) offers a consensus view of future market volatility. This broad agreement among traders and investors about what is expected to happen is particularly useful for making moves in a portfolio that aim to respond to or anticipate market changes.

Another advantage of the VIX is the variety of financial products related to it, such as futures and options, which allow for different strategies to either protect against or benefit from market changes. This flexibility is not as readily available with other volatility indexes.

Historical events have proven the VIX's worth (see section 1.6.5). These instances validate its reliability in forecasting[9] market trends, reinforcing why it is favored for adjusting investment strategies to either take advantage of or protect against market shifts (Lehtonen 2023).

In summary, the VIX stands out for its real-time forecast of market volatility. On paper, it is a good forward-looking forecast with a track record during significant market events, making it a valuable indicator.

1.6.2 Current Calculation of the VIX Values

The VIX index calculates volatility using standard monthly S&P 500 options expiring on the third Friday and weekly S&P 500 options expiring on other Fridays. It includes S&P 500 options expiring between 23 and 37 days from the calculation date (CBOE 2019). The formula aggregates weighted prices of S&P 500 puts and calls across various strike prices. Eligible options must have valid bid and ask prices indicating market expectations of which strike prices will be reached by the underlying before expiry (Preston, Edwards 2017). The detailed calculation involves complex mathematics – However the theory of the calculation is the following (streamlined explanation abstracting the specific mathematical details and focusing on the logical flow of the calculation process) :

I. **Selection of Options :** Use out-of-the-money S&P 500 call and put options near the at-the-money strike price, excluding those with zero bids.

II. **Calculation of Individual Option Contributions :** Determine each option's contribution based on its strike price, bid/ask prices, and time to expiration.

III. **Aggregation and Final Calculation :** Combine the contributions to compute the VIX, representing the market's expectation of 30-days forward-looking volatility.

1.6.3 Understanding the VIX Values (Preston, Edwards 2017)

The VIX, known for its strong inverse relationship with stock market returns, tends to rise when the S&P 500 falls, reflecting increased investor fear. Conversely, a falling VIX suggests a calmer market and less concern. It is crucial to understand that trading based on volatility does not directly predict market declines, as the market can drop even during periods of low volatility.

[9] Refer to Part II of this thesis which provides an in-depth analysis of the VIX predictability of markets movements.

Volatility measures how much and how often market prices change, not the actual price of the asset. It looks at the size and frequency of price movements, regardless of whether prices go up or down. This is why VIX values are expressed in percentage points. Typically, a VIX value under 12 signals low market volatility expectations and a VIX value under 20 is associated with market stability, whereas values above 30 are seen as indicative of high volatility (Fidelity International 2019).

The VIX is often regarded as a forecaster of market peaks and troughs: very high values may signal potential upward movement in the S&P 500, while very low values might suggest a downturn.

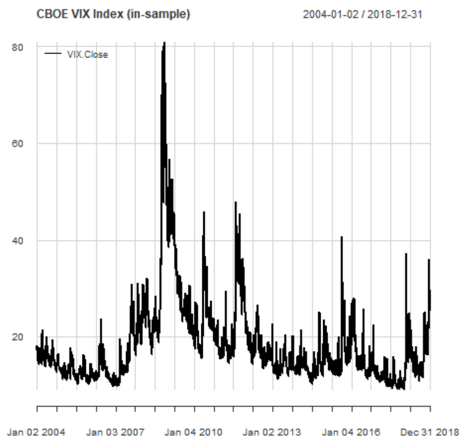
Notably, the VIX often carries a “premium,” meaning it tends to predict higher volatility than what materializes in the ensuing 30 days. This premium, which averages around 4 to 5 percentage points above realized volatility, underscores the importance of not directly equating VIX levels with expected market volatility percentages (Lehtonen 2023).

1.6.4 Analysis of VIX Movements During the In-Sample Period

The VIX exhibited significant fluctuations during the in-sample period from January 2004 to December 2018, which were majorly influenced by economic events and shifts in market sentiment. During the mid-2000s, the VIX remained relatively low (period of market stability). With the 2008 Great Recession, the VIX spiked to historically high levels. Following the crisis, the VIX gradually declined as markets stabilized. This was emphasized by aggressive monetary policy measures from central banks globally. However, the VIX spiked again during various events such as the European debt crisis in 2011 and the US debt ceiling debates, which introduced significant risk aversion and volatility into markets.

Throughout the remainder of the period, the VIX spiked during periods of geopolitical tensions, economic data surprises and shifts in expected monetary policy (for e.g., significant VIX movements were captured during the 2015 Chinese stock market crash and the 2016 US presidential elections). Each spike in the VIX often corresponded with downturns in the S&P 500.

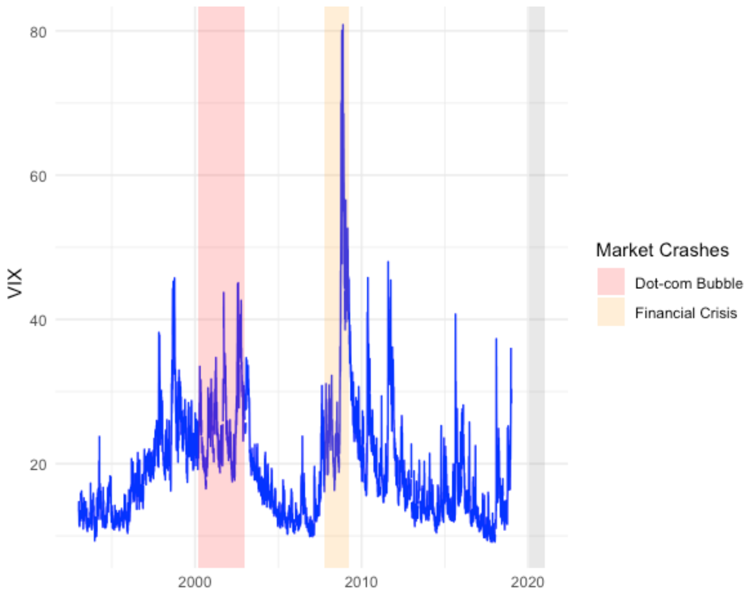
Figure 6 - CBOE VIX Evolution (In-Sample)



The Figure 6 help encapsulate the behavior of the VIX, highlighting its tendency to cluster within certain ranges during stable market conditions, and to spike during periods of financial uncertainty or market stress.

1.6.5 VIX During Crisis (2000 “The dot-com bubble” and 2008 “The Great Recession”)

Figure 7 - VIX During Crisis (In-Sample)



The dot-com bubble (Hayes 2023) occurred between the late 1990s and early 2000s. This period was characterized by a rapid rise in equity markets fueled by investments in internet-based companies. The bubble burst in March 2000, led to a dramatic market decline. During pre-burst period (1998-2000), the VIX was relatively low and stable, averaging around 20. This period saw significant valuations of tech stocks, driven by speculative investments. As the bubble burst, the VIX spiked, exceeding 40 at various points. Post-burst (2001-2002), the VIX remained elevated. This sustained high volatility period was due to market adjustments, corporate scandals (such as Enron[10]) (Bondarenko 2024), and overall economic uncertainty.

The 2008 financial crisis (Manda 2010) catalyzed a significant surge in market volatility, with the S&P 500 losing about 56% of its value from its peak in October 2007 to the trough in March 2009. Concurrently, the VIX index has more than tripled. This stark rise in the VIX highlighted the intense fear and uncertainty among investors during the crisis. The period was marked by a notable increase in the trading of VIX futures, reflecting their growing importance as a hedging mechanism against market declines. These futures contracts, representing market predictions of future VIX levels, saw a surge in both volume and open interest, indicating a heightened demand for volatility-based protection.

1.6.6 Limitations of the VIX

Despite its popularity, the VIX is often criticized for its limited predictive power of future market volatility. Studies have shown that the daily movements of the VIX are highly correlated with past S&P 500 returns, indicating that much of the information the VIX provides is already reflected in the market, suggesting that its ability to predict future volatility is constrained (Hait 2017).

In Part II of this thesis, we will delve deeper into the predictive power of the VIX.

Another limitation of the VIX is the contango phenomena in the futures market. Contango occurs when the futures prices are higher than the expected spot price, leading to a situation where longer-term futures contracts are more expensive than shorter-term contracts. This is typically observed during low-volatility periods, causing products based on VIX futures (like the VXX – "*iPath Series B S&P 500 VIX Short-Term Futures ETN*" or the UVXY – "*ProShares Ultra VIX Short-Term Futures ETF*) to bleed value over time as they roll futures contracts forward, leading to consistent losses if the VIX level does not increase (Butler 2022).

Conversely, backwardation happens when the VIX futures price is lower than the spot price, often occurring during periods of high volatility. This can lead to gains for long volatility products, but these high volatility periods are usually quite short, making it challenging to profit from this situation (Butler 2022).

Since the strategy (detailed in Part III), does not rely on rolling VIX futures contracts, it avoids the value erosion issues related to contango.

[10] Enron Scandal : Series of events (accounting fraud) that resulted on the 2001 bankruptcy of the company Enron Corporation and the dissolution of the firm Arthur Andersen.

1.6.7 Analysis of the Relation Between the VIX and Other Asset Classes

Figure 8 - Log Returns of the VIX vs. Major Asset Classes (In-Sample)

Figure 8 shows the daily log returns of the VIX and major indices that have been chosen to highlight the relation between VIX levels and other asset classes (global equities, global fixed income, commodities and gold).

VIX (Volatility Index) and Global Equities (ACWI):

There is a noticeable inverse relationship between the VIX and the ACWI index. When the VIX spikes, indicating increased market volatility and fear, the returns of the ACWI often show significant drops.

- **Interpretation:** Higher market volatility (higher VIX) usually corresponds with declining global equity prices. This is expected as increased fear and uncertainty in the market lead to sell-offs in equities.

VIX and Global Fixed Income (HAF.TO):

The HAF.TO index shows much less volatility compared to the VIX. During periods when the VIX spikes, the HAF.TO exhibits relatively minor fluctuations.

- **Interpretation:** Fixed income securities are generally less volatile than equities and tend to provide stability during market turbulence. Investors often overweight bonds during high volatility periods for safety, which can moderate price changes in this asset class.

VIX and Gold (GLD):

The GLD index shows increased volatility during periods when the VIX spikes. However, the direction of gold's returns is not always inverse to the VIX; sometimes gold prices rise, and other times they fall.

- **Interpretation:** Gold is often considered a safe-haven asset. During periods of market stress (high VIX), investors may move to gold, increasing its volatility. The direction of gold's price change during these periods can depend on various factors, including investor sentiment and market conditions.

VIX and Commodities (DBC):

The DBC index shows increased volatility during periods when the VIX spikes. The returns for commodities become more volatile, like equities, during high-stress periods.

- **Interpretation:** Commodities are influenced by both market sentiment and global economic conditions. During periods of high volatility (high VIX), the uncertainty can lead to significant price swings in commodities as market participants react to changing economic forecasts and risk perceptions.

1.7 Methodology and Summary

The methodology for Part I of the thesis is structured to set the scene for further analysis and development of Volatility-Based DAA strategies. It involves the following :

1. **Literature Review:** Review of academic and industry literature to understand the current state of financial markets, the role of volatility and the VIX mainly, as well as the composition and current state of the S&P 500.

2. **Market Analysis and Historical Market Context:** The analysis and resulting graphs were executed in R, via the collection of market data from credible sources.

3. **Portfolio Management Strategies:** Examination of various portfolio management strategies, including passive strategies and DAA models.

4. **Analysis of the S&P 500:** Analysis of the S&P 500, its correlation to major indices, its composition, and cycles. This included a historical empirical performance analysis and statistical metrics analysis.

5. **Volatility Analysis:** Analysis of volatility, types of volatility and differences, VIX, its calculation and composition. This included a historical empirical analysis, reaction to crises, other volatility indicators, and limitations of the VIX, and its relation to other asset classes.

Summary

Part I of this thesis establishes a foundation for further analysis and models development, supporting the rationale for the proposed volatility-based DAA strategies. It provides an understanding of current market dynamics, with mid-2024 financial markets showing a mix of optimism and caution, driven by the S&P 500's strong performance in 2023 due to easing concerns over interest rates and inflation.

The U.S. economy's stability, marked by a balanced labor market and moderated core PCE inflation, is complemented by the Federal Reserve's cautious approach to rate cuts, reducing near-term recession risks. Global growth is moderating but firming up, driven by European consumers and global capEx, despite persistent core inflation around 3% due to tight labor markets and wage gains.

Volatility remains key, with the VIX offering valuable insights for managing portfolio risk and dynamically adjusting asset allocations. The promising DAA strategy based on the VIX aims to navigate market fluctuations, enhance returns, and mitigate risks. Analyzing the S&P 500's sector composition shows dominance in Information Technology, Health Care, and Financials, with cyclical sectors sensitive to economic cycles and defensive sectors providing stability. The high correlation with global indices highlights the U.S. market's significance in global equity performance. Historical analysis during significant financial events underscores the importance of understanding market volatility's impact on portfolio performance.

2. PART II – EMPIRICAL STUDY: VIX PREDICTABILITY

The purpose of this Part II is to introduce the decomposition of the VIX using an empirical approach and R codes. Initially, it is demonstrated that the VIX is primarily composed of an element that reflects realized volatility (whether conditional or not). Contrary to its reputation and its common association as the measure of volatility under which option hedging activities for one-month maturities should be conducted according to the Black & Scholes model, the VIX is essentially made up of a reactive component. It is only the secondary component of the VIX, known as the **Variance Premium** (comments and analyses in Part III), which might be interpreted as containing anticipations other than those based on current volatility.

This introduction sets the stage for seminal works by French et al. (1987), Bollerslev et al. (2009), and the extensive literature on the decomposition of the VIX that has developed since 2010.

2.1 Data

2.1.1 Evolution of the VIX Methodology

The VIX was created in 1993 by the CBOE. Initially, it was designed to measure the market's expected volatility by looking at at-the-money S&P 500 options with a one-month maturity. Since January 1, 2004, the VIX has been calculated using an aggregation of implied volatilities from both puts and calls across various strikes, also with a one-month maturity on the S&P 500 (detailed under section 1.6.2). Therefore, data prior to 2003 reflects the old methodology.

2.1.2 Data Sample

Daily data for the S&P 500 and the VIX from January 2004 to the end of December 2018 were downloaded from Yahoo Finance using R. To calculate a rolling volatility of 252[11] days on the S&P 500 and to apply a GARCH model ex-ante[12] on 2500 data points, data from January 1993 to December 2018 were downloaded for the stock index.

To test the DAA strategies, our out-sample backtest period is **from January 2019 to December 2023**. Therefore, all descriptive statistical analyses and estimations have been conducted on periods **before** the out-sample period start to avoid any bias in our strategies final backtest.

[11] Calculating the volatility of the S&P 500 index over a rolling window of 252 trading days helps capture the changing volatility regime of the market over time.
[12] GARCH stands for *Generalized Autoregressive Conditional Heteroskedasticity*. It is a statistical model used to analyze and forecast volatility in financial time series data. Applying a GARCH model ex-ante means using the model to predict future volatility based on historical data.

Table 1 provides the number of daily data points available in-sample and their distribution. Additionally, both samples show a median and modal number of trading days per month at 21.

Table 1 - Trading Days Distribution (In-Sample)

	In-sample
Number of trading days	3775
Pecentage of the total sample	74.1
Median of trading days number per month	21
Average of trading days number per month	20.97

2.2 Comparison Between the VIX and Other Volatility Measures

The VIX is commonly understood as a reflection of volatility according to the Black-Scholes model over the next 21 days, primarily intended to facilitate dynamic delta hedging. This hedging approach aims to adjust continuously in response to market movements, ideally allowing neither gains nor losses (thus maintaining an arbitrage-free condition). This theoretical framework ensures that the VIX serves not just as a measure of expected market volatility, but as a tool for managing portfolio risk through precise adjustments to option positions.

A practical comparison to this model might involve historical volatility associated with hedging activities. For instance, one could calculate the volatilities of 5-minute return intervals for each of the upcoming 21 days of coverage, and then average these values. Such analysis would provide a granular view of short-term fluctuations, contrasting with the broader market predictions embodied by the VIX.

However, this approach requires extensive data and is time-consuming. Moreover, for practitioners, the volatility calculated using simple close-to-close data is the one they experience if they are not directly involved in trading activities. Therefore, subsequent comparisons will be conducted using volatility measures based on this type of data.

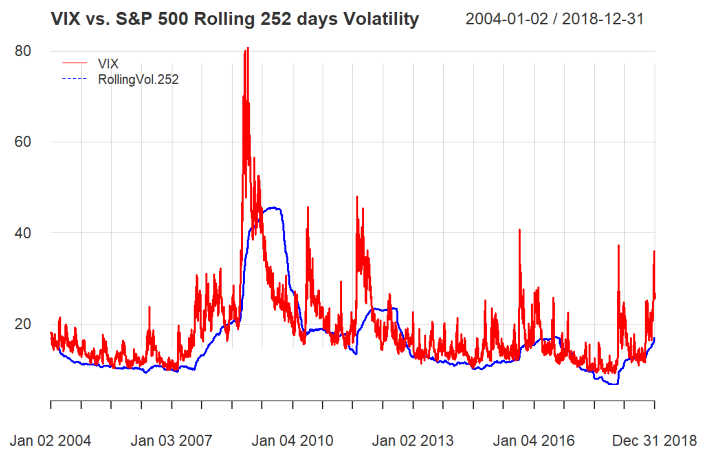
2.2.1 VIX vs. Rolling 252-day Volatility

A preliminary comparison can be made between the VIX and the historical volatility calculated over the past 252 trading days. This approach is utilized, for example, in the S&P 500 Dow Jones Global Indices (2007). As previously noted, the implied volatility reflected by the VIX pertains to options with a one-month maturity, suggesting that extending the time window beyond perhaps two trading days may not be appropriate.

Two types of comparisons can be conducted:

Firstly, by comparing at a given time t the available values of the VIX and the rolling volatility calculated at that moment with the last 252 observations. Figure 9 illustrates this scenario and indicates that historical volatility is generally lower, but it suggests that the VIX might serve as a leading indicator. Indeed, the increase in volatility from 2006 up to the 2008 Great Recession was anticipated by the VIX, as well as the decrease observed in 2010.

Figure 9 - Rolling 252-day Volatility vs. VIX (In-Sample)



Secondly, by comparing the VIX at time t with the realized future volatility, calculated at t+T. It would be reasonable to set T=21 days, allowing for a comparison between the implied volatility and the realized volatility calculated 21 days later (see figure 10).

Figure 10 - Rolling 252-day Future (T=21) Volatility vs. VIX (In-Sample)

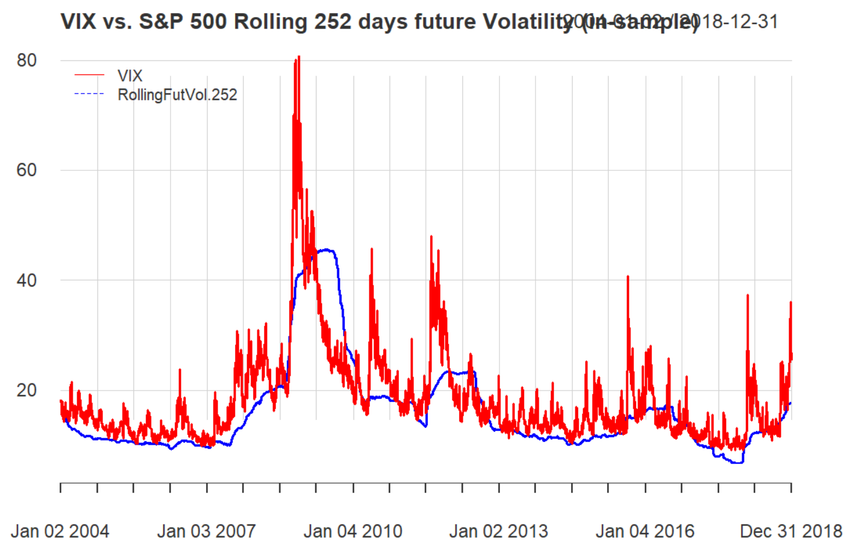
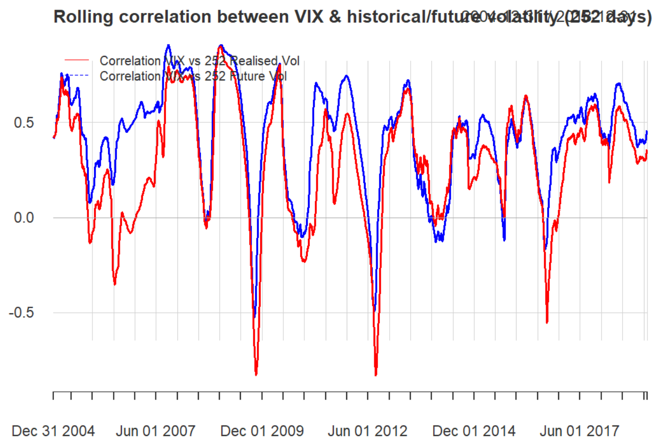


Figure 11 compares the rolling correlations (calculated over 252 days) between two pairs of volatilities (historical versus implied and realized versus implied). It is notable that, with few exceptions, the relationship between implied volatility and future volatility is consistently more significant. This observation seems to confirm the predictive nature of implied volatility. However, the figure also shows numerous periods where the rolling correlations are very weak or even negative.

Figure 11 - Rolling Correlation (252-day) between Implied Volatility and Historical Volatility (calculated over 252 days) / Future Volatility (In-Sample)

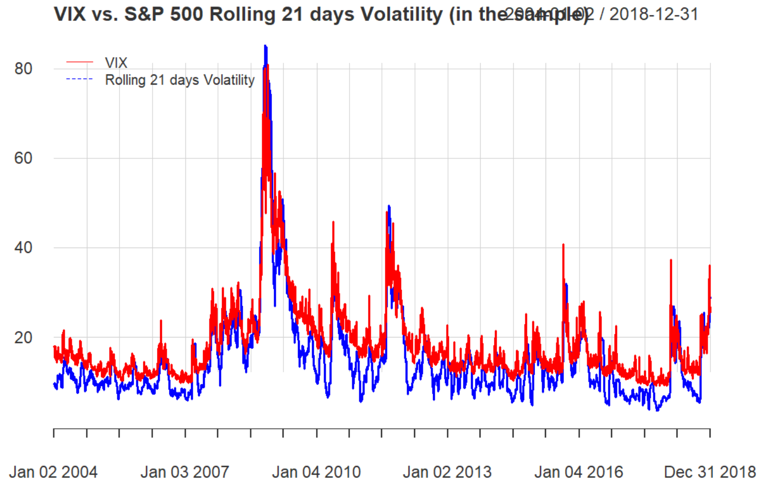


The behaviour of the rolling correlations casts doubts on the anticipatory power of implied volatility. Could the perception of predictive aspects of volatility merely be attributed to the “Ghost Effect” combined with a one-year time window, indicating a diminished sensitivity to older data? This observation motivates the investigations that follow. We will compare the volatility with volatilities calculated based on past returns of the S&P 500 but using models and methodologies that allow greater sensitivity to recent data.

2.2.2 VIX vs. Rolling 21-day Volatility

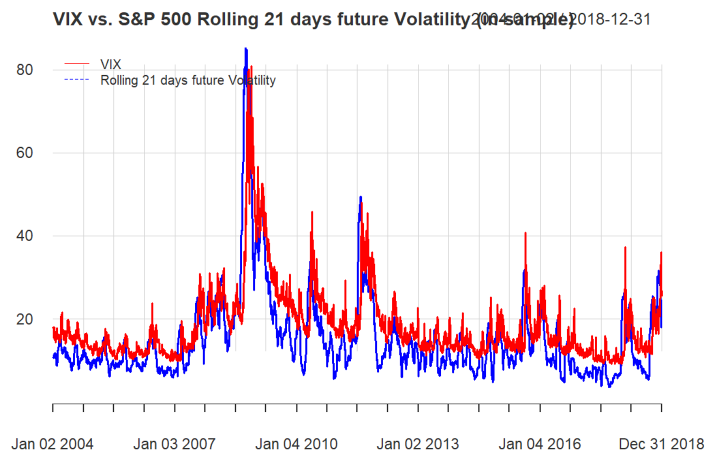
A straightforward approach involves calculating historical volatility using equally weighted returns over 21 days. The validity and robustness of an estimate based on merely 21 daily data points could be questioned. However, if these approaches may seem questionable from a theoretical point of view, they will nevertheless demonstrate a strong correlation with the VIX. Figure 12 first demonstrates that this indicator is highly synchronized with historical volatility, registering a correlation of 0.91 with implied volatility across the in-sample dataset.

Figure 12 - Historical Volatility (calculated over 21 days) and Implied Volatility (In-Sample)



The comparison with future volatility shown in figure 13 indicates a weakened link between the indicators. It appears that future volatility tends to lead implied volatility, suggesting an inverse relationship in terms of anticipation.

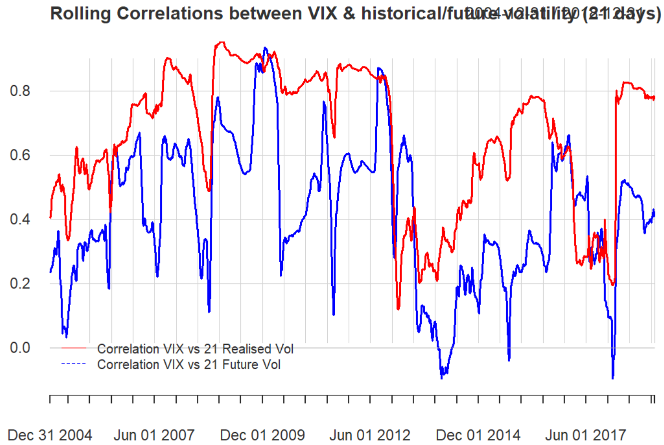
Figure 13 - Future Volatility (calculated over 21 days, shifted by 21 days) and Implied Volatility (In-Sample)



The correlation between implied and future volatility drops to 0.78 compared to the historical volatility correlation of 0.91. This reduction occurs gradually when introducing a (minus) one-day lag[13]. As illustrated in figure 14, the rolling correlations of implied volatility with historical and future volatilities show that correlations with historical volatility are almost always higher. Moreover, the rolling correlation between implied volatility and historical volatility is rarely below 0.4 (and never negative) and most often above 0.6.

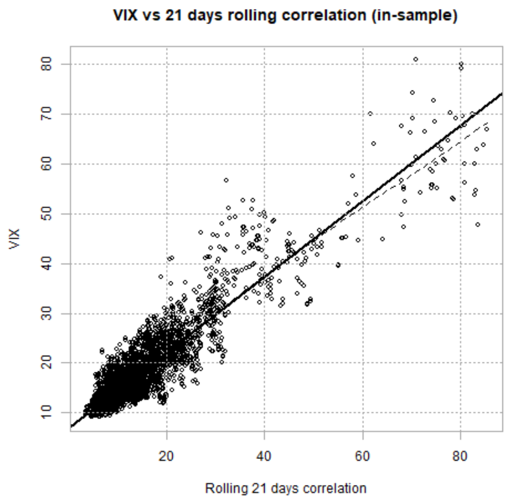
[13] See Table A04 in Appendix 2. The decrease in correlation occurs after 4 days ahead.

Figure 14 - Rolling Correlations (252 days) between Implied Volatility and Historical Volatility (calculated at 21 days) / Future Volatility (In-Sample)



Thus, a significant portion of implied volatility indeed stems from the recent volatility of the data. A linear regression of implied volatility on 21-day historical volatility yields[14] an intercept of 6.97 and a slope of 0.78. All parameters are significant, and 84% of the variance of implied volatility is explained by historical volatility.

Figure 15 - Linear Regression of Implied Volatility on 21-day Historical Volatility (In-Sample)



2.2.3 VIX vs. Realized GARCH Volatility

While the implied volatility extracted has been directly linked to recent historical volatility, the 21-day historical volatility has several limitations in capturing recent market volatility. First, the predictive aspect of volatility during the 21-day dynamic hedging period of the option is ignored, making the number of days somewhat arbitrary. Additionally, historical volatility suffers from a "Ghost Effect" that can bias the measurement of recent volatility.

[14] See Table A05 in Appendix 2 for regression estimations.

Since Mandelbrot's studies in 1963, it has been well-known that financial data exhibit volatility clusters. For these reasons, an approach based on conditional variance as modelled by the GARCH family of models seems more appropriate (for a more detailed presentation of GARCH models, see Alexander (2008) or Hull (2023)).

Three types of models will be used going forward:

1) Normal GARCH (1,1) model

The conditional variance equation is as follows:

Equation 2 - Conditional Variance

$$\sigma_t^2 = \omega + \alpha \cdot e_{t-1}^2 + \beta \cdot \sigma_{t-1}^2 \; with \; e_t = \varepsilon_t \cdot \sigma_t \; and \; \varepsilon_t \sim N(0,1)$$

The error term (alpha) measures the conditional volatility's response to a market shock. If alpha is large (above 0.1), the volatility is highly sensitive to market events. The lag parameter (beta) measures the persistence of past shocks. If beta is substantial (greater than 0.99), the variance induced by a financial crisis will take time to fade. The sum of alpha and beta measures the rate of convergence towards long-term volatility. When this rate exceeds 0.99, the term structure of variance expectations is flat. This model can be seen as reverting to the long-term conditional volatility measured by the ratio of omega divided by (*1 - alpha - beta*).

The below equation provides the one-day variance prediction, which allows to recursively build a term structure of variance predictions:

Equation 3 - One-day Variance Prediction

$$\hat{\sigma}_{t+s+1}^2 = \hat{\omega} + \left(\hat{\alpha} + \hat{\beta}\right) \cdot \hat{\sigma}_{t+s}^2$$

2) Skewed Student GARCH (1,1) model

The below equation allows to reflect the fat tails and the asymmetry of the distribution of returns and of conditional variance:

Equation 4 - Fat Tails and Asymmetry Reflection

$$\sigma_t^2 = \omega + \alpha \cdot e_{t-1}^2 + \beta \cdot \sigma_{t-1}^2 \; with \; e_t = \varepsilon_t \cdot \sigma_t \; and \; \varepsilon_t \sim St(0,1,\nu)$$

3) GJR-GARCH (1,1) model

This model includes asymmetries, acknowledging that volatility increases are stronger after losses than after gains. This model type is particularly suitable for stocks (after a loss, the equity-to-debt ratio increases, thus increasing the company’s leverage and vulnerability to crisis). The conditional variance equation becomes:

Equation 5 - GJR-GARCH Conditional Variance

$$\sigma_t^2 = \omega + \alpha \cdot e_{t-1}^2 + \beta \cdot \sigma_{t-1}^2 + \gamma \cdot e_{t-1}^2 \ (e_{t-1} < 0) \ with \ e_t = \varepsilon_t \cdot \sigma_t \ and \ \varepsilon_t \sim St(0,1,\nu)$$

The long-term volatility is the ratio (*omega* + *lambda2* * *alpha*) divided by (*1- alpha – beta*). The one-day prediction will be:

Equation 6 - GJR-GARCH One-day Variance Prediction

$$\hat{\sigma}_{t+s+1}^2 = \widehat{\omega} + \left(\hat{\alpha} + \hat{\beta} + 0.5 \cdot \hat{\gamma}\right) \cdot \hat{\sigma}_{t+s}^2$$

The comparison between variance models will first be conducted ex-post for the GARCH models, using the entire in-sample dataset (with extra data of 2002 for a relevant volatility estimation as of January 2003). Subsequently, we will adopt the perspective of a dynamic portfolio manager, using the previous 2500 returns at a given time t to calculate variance. Note that this second approach requires significantly more computation time, as the GARCH models must be estimated daily within the in-sample period, and the process is repeated the following day.

2.2.3.1 Ex-Post In-Sample Analysis

Table 2 presents the results of the GARCH model estimations performed using the rugarch package in R. All parameters are significant for the Normal GARCH model. The estimated values align with expectations; notably, the sum of alpha and beta is less than 1, ensuring the stability of the process. An alpha greater than 0.1 indicates that conditional volatility is highly sensitive to market events, while a beta less than 0.9 suggests low persistence of conditional volatility independent of market events. The long-term annualized volatility[15] is estimated at 17.6%. One or two parameters for the other models are not significantly different from zero.

Table 2 - GARCH Model Estimation (In-Sample)

	omega	alpha1	beta1	skew1	shape	gamma1
GARCHN1-1 coefficients	0.00020%	0.1044	0.8794	NA	NA	NA
GARCHN1-1 t-stat	2.42	9.74	77.30	NA	NA	NA
GARCHS1-1 coefficients	0.00010%	0.1045	0.8923	0.9122	6.5179	NA
GARCHS1-1 t-stat	1.16	6.69	62.97	50.80	13.87	NA
GARCHGJR1-1 coefficients	0.00020%	0	0.8874	0.8777	7.3466	0.2019
GARCHGJR1-1 t-stat	1.74	0.00	64.41	48.67	8.34	6.95

Table 3 provides the correlation matrix between implied volatility, four different volatility measures and the average volatility predictions for the next 21 days using the term structures derived from each of the three GARCH models. First, it is observed that, the

[15] Please note that the estimations are for the daily variance and that inputs are not expressed in points. Thus, omegas are small and conditional long term should be transformed in annualized volatility.

correlations between the VIX and the predictions are all lower than those calculated with the GARCH volatility (which is a one-day prediction). This insight further suggests that while the VIX incorporates elements of forward-looking information, its immediate past volatility plays a more significant role in its calculation than the anticipated future volatility.

Then, the strongest correlation with implied volatility is with the Student GARCH model (0.924), followed by Normal GARCH (0.92), the historical volatility over 21 days (0.915), and the volatility from the GJR GARCH model. As the differences between these correlations are relatively minor, given the estimation results, the focus will remain primarily on the estimations of the Normal GARCH model, with the others provided for reference.

Table 3 - Correlations between VIX, Historical Vol. & Ex-Post GARCH Volatility (In-Sample)

	VIX	R.21V	N GARCH	S GARCH	GJR GARCH	N GARCH - pred	S GARCH - pred	GJR GARCH -pred
VIX	1.000	0.915	0.920	0.925	0.911	0.918	0.924	0.910
Rolling 21 days Volatility	0.915	1.000	0.977	0.981	0.938	0.975	0.981	0.938
Normal Garch(1,1) Volatility	0.920	0.977	1.000	0.999	0.972	0.999	0.999	0.973
Student Garch(1,1) Volatility	0.925	0.981	0.999	1.000	0.969	0.998	1.000	0.969
GJR Garch(1,1) Volatility	0.911	0.938	0.972	0.969	1.000	0.971	0.969	1.000
Normal Garch(1,1) average Volatility forecast	0.918	0.975	0.999	0.998	0.971	1.000	0.999	0.972
Student Garch(1,1) average Volatility forecast	0.924	0.981	0.999	1.000	0.969	0.999	1.000	0.969
GJR Garch(1,1) average Volatility forecast	0.910	0.938	0.973	0.969	1.000	0.972	0.969	1.000

Furthermore, there is a very strong correlation between the 21-day historical volatility and that of the Normal GARCH (0.977), not to mention the Student GARCH model (0.981). For asset management practitioners, these strong correlations raise questions about the utility of these complex models compared to a simple approach that could be implemented on Excel and easily understood.

Table 4 points out that a 17-days rolling volatility registers a correlation with the Normal GARCH model at 0.983, the highest level for a historic rolling volatility, while with the exponential volatility of RiskMetrics (with an exponential weighting of 0.94), the correlation increases to 0.977. As noted by Hull (2023), the model with exponential weighting is a special case of the Normal GARCH(1,1), with β equal to the λ parameter of the exponential weighting (and $\alpha + \beta = 1$). Here, instead of using exponential volatility with a lambda of 0.94, one could calculate another with $\lambda = 0.8794$. The correlation with the GARCH model is the strongest, recorded at 0.987.

Table 4 - Between VIX, Normal GARCH and Different Time Windows Rolling Volatility (In-Sample)

	VIX	N GARCH	EMV.0.94	EMV.beta
VIX	1.0000	0.9202	0.9436	0.9300
garchvolxp.n.1.1	0.9202	1.0000	0.9771	0.9873
EMV.lambda.0.94	0.9436	0.9771	1.0000	0.9802
EMV.lambda.beta	0.9300	0.9873	0.9802	1.0000
Rolling.10.days.volatility	0.8917	0.9713	0.9427	0.9789
Rolling.11.days.volatility	0.8971	0.9755	0.9498	0.9817
Rolling.12.days.volatility	0.9009	0.9779	0.9557	0.9838
Rolling.13.days.volatility	0.9039	0.9801	0.9609	0.9851
Rolling.14.days.volatility	0.9060	0.9819	0.9654	0.9855
Rolling.15.days.volatility	0.9076	0.9827	0.9693	0.9853
Rolling.16.days.volatility	0.9086	0.9828	0.9726	0.9845
Rolling.17.days.volatility	0.9098	0.9823	0.9754	0.9832
Rolling.18.days.volatility	0.9114	0.9812	0.9778	0.9816
Rolling.19.days.volatility	0.9125	0.9798	0.9798	0.9798
Rolling.20.days.volatility	0.9136	0.9783	0.9814	0.9778
Rolling.21.days.volatility	0.9145	0.9765	0.9828	0.9755
Rolling.22.days.volatility	0.9147	0.9744	0.9839	0.9729

	VIX	N GARCH	EMV.0.94	EMV.beta
Rolling.23.days.volatility	0.9155	0.9721	0.9848	0.9704
Rolling.24.days.volatility	0.9158	0.9696	0.9855	0.9680
Rolling.25.days.volatility	0.9161	0.9675	0.9860	0.9653
Rolling.26.days.volatility	0.9164	0.9650	0.9863	0.9625
Rolling.27.days.volatility	0.9168	0.9624	0.9863	0.9594
Rolling.28.days.volatility	0.9172	0.9594	0.9863	0.9566
Rolling.29.days.volatility	0.9176	0.9565	0.9862	0.9538
Rolling.30.days.volatility	0.9175	0.9539	0.9859	0.9512
Rolling.31.days.volatility	0.9174	0.9513	0.9855	0.9485
Rolling.32.days.volatility	0.9172	0.9488	0.9850	0.9457
Rolling.33.days.volatility	0.9168	0.9460	0.9844	0.9430
Rolling.34.days.volatility	0.9165	0.9433	0.9837	0.9403
Rolling.35.days.volatility	0.9162	0.9407	0.9830	0.9378
Rolling.36.days.volatility	0.9160	0.9383	0.9821	0.9353
Rolling.37.days.volatility	0.9158	0.9358	0.9812	0.9327
Rolling.38.days.volatility	0.9156	0.9333	0.9802	0.9301
Rolling.39.days.volatility	0.9153	0.9308	0.9792	0.9275
Rolling.40.days.volatility	0.9148	0.9283	0.9782	0.9251

Table 4 also shows that the exponential volatility with a lambda of 0.94 has the highest correlation with the VIX, which may indicate how the seemingly arbitrary choice of 0.94 was made by RiskMetrics. Additionally, it is observed that the highest rolling correlation with the VIX occurs at 29 trading days, which is just under a month and a half in calendar days.

Figure 16 illustrates the rolling correlations between implied volatility and, a year later, the volatility from the Normal GARCH model and the volatility calculated over 29 days. It is first observed that the trend of the correlation for historical volatility is very similar whether the calculations are conducted over 21 or 29 days. Moreover, the correlation with the Normal GARCH volatility is almost always higher, but more importantly, it is less volatile and demonstrates a certain capacity to reduce drops in correlation.

Figure 16 - Rolling Correlations (252 days) between Implied Vol. and Normal GARCH Volatility / 29 days Rolling Volatility (In-Sample)

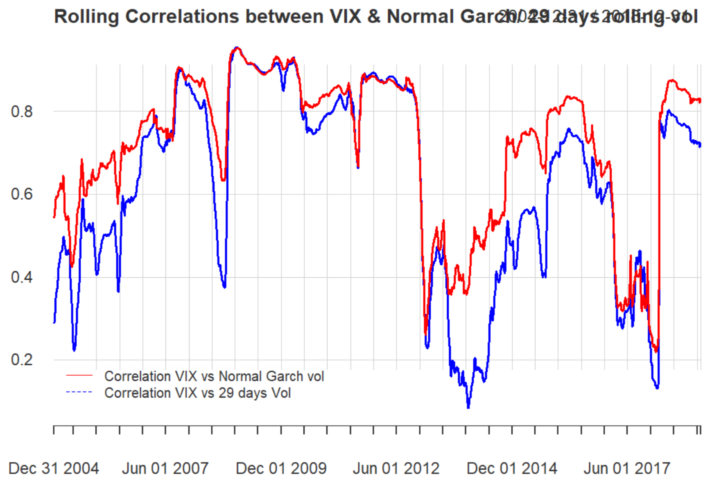
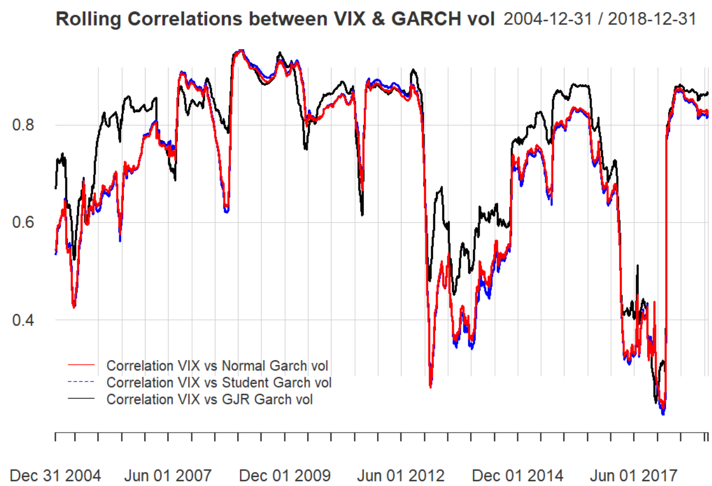


Figure 17 indicates that the volatility trends of the GARCH models are relatively similar between the Normal GARCH and the Skewed Student GARCH models. This similarity suggests that despite the differing assumptions and structures of these models, their performance in terms of tracking or predicting market volatility shows consistent patterns.

Figure 17 - Rolling Correlations (252 days) between Implied Volatility and Normal / Skewed Student / GJR GARCH Volatility (In-Sample)



2.2.3.2 Ex-Ante In-Sample Analysis

We now consider the realistic scenario where GARCH model estimations are performed using only the available data. This approach involves recalculating the GARCH models daily using the last 2500 data points.

Table 5 presents the correlation matrix of the volatilities obtained with this ex-ante approach, compared to the previously conducted ex-post analysis. The ex-ante correlations are slightly higher, suggesting that the underlying process is genuinely conditional. This means that the volatilities depend not only on current fluctuations but also, though weakly, on their entire history. However, the differences remain minor. The correlation with the ex-ante Normal GARCH volatility is 0.931, compared to 0.920 with the ex-post. Given the computation time required to generate the ex-ante series (several hours for each series in our case), the prior analysis using ex-post data is clearly justified.

Table 5 - Correlations between VIX, Historical Volatility, Ex-Ante & Ex-Post GARCH Volatility (in-sample)

	VIX	21 days Vo	29 days Vo	trics EW vo	te Normal G	te Student	st Normal G	st Student G
VIX	1.000	0.915	0.918	0.944	0.931	0.925	0.920	0.925
Rolling 21 days Volatility	0.915	1.000	0.980	0.983	0.981	0.956	0.977	0.981
Rolling 29 days Volatility	0.918	0.980	1.000	0.986	0.973	0.939	0.957	0.965
RiskMetrics EW volatility	0.944	0.983	0.986	1.000	0.988	0.961	0.977	0.983
Ex ante Normal Garch	0.931	0.981	0.973	0.988	1.000	0.978	0.993	0.996
Ex ante Student Garch	0.925	0.956	0.939	0.961	0.978	1.000	0.979	0.978
Ex post Normal Garch	0.920	0.977	0.957	0.977	0.993	0.979	1.000	0.999
Ex post Student Garch	0.925	0.981	0.965	0.983	0.996	0.978	0.999	1.000

2.3 Methodology and Summary

The methodology for Part II of the thesis is structured to introduce the decomposition of the VIX using an empirical approach and R codes. It involves the following:

- **Literature Review:** Introduction to seminal works by French et al. (1987), Bollerslev et al. (2009), and other literature on VIX decomposition since 2010.
- **Comparative Analysis:** Comparison between the VIX and other volatility measures, including rolling 252-day volatility, rolling 21-day volatility and realized GARCH volatility.
- **Further Analyses:** Comparison between the VIX and GARCH Models: Application of Normal GARCH (1,1), Skewed Student GARCH (1,1), and GJR-GARCH (1,1).

Summary

Part II of this thesis demonstrates that the VIX is mainly composed of realized volatility, with the Variance Premium[16] providing some forward-looking insights. The analysis shows that while the VIX is used as a measure of expected market volatility, it is heavily influenced by recent historical volatility, making its predictive power limited. Comparing the VIX with rolling 252-day historical volatility indicated that the VIX often acts as a leading indicator, while the 21-day historical volatility showed a stronger correlation with

[16] Further described in Part III.

the VIX, highlighting its sensitivity to short-term movements. Notably, the highest correlation between the VIX and historical volatilities was found with 17-day rolling volatility and exponential volatility with a lambda of 0.94 (the lambda represents the speed at which the impact of older data decays).

The empirical analysis of GARCH models reinforces the finding that the VIX is closely linked to recent historical volatility. Among the models tested, the Normal GARCH exhibited the highest correlation with the VIX, followed by the Skewed Student GARCH. This suggests that more complex models do not substantially outperform simpler historical volatility measures when it comes to tracking the VIX.

Ex-ante analysis, which examines future volatility estimates, showed only marginal improvements over ex-post analysis, highlighting that ex-ante models offer limited practical advantages over simpler methods.

In general, rolling correlations between the VIX and various volatility measures demonstrated that the Normal GARCH model maintained the most stable correlation with the VIX, particularly in periods of high volatility. This stability highlights the VIX's strong dependence on recent volatility. Although GARCH models are designed to capture volatility clustering, their additional complexity provides only slight gains in predictive power over more straightforward historical volatility calculations.

These findings highlight the importance of integrating realized and implied volatility, especially through the Variance Premium, in developing DAA strategies. The VIX’s role as a reactive indicator with some predictive capacity helps in managing portfolio risk. Insights from this study help refine volatility-based DAA strategies in Part III to optimize returns and manage risks in varying market conditions.

3. PART III – IMPLEMENTING THE VOLATILITY-BASED DAA

To assess if a DAA strategy based on the VIX or the Variance Premiums can improve a fund's performance and bolster risk management, we will create various models in R. Our approach focuses on leveraging simplified models rather than relying on complex statistical models and algorithms for perfect optimization. These simplified models can be effectively marketed to private investors in a wealth management context, offering a passive DAA based on volatility metrics such as the VIX itself or the Variance Premium, as introduced in Part II and further described in the following sections.

We have several strategies to choose from, such as mimicking an existing fund while applying our new DAA rules via an overlay strategy — utilizing options, futures, ETFs, or swaps — or crafting a complete ETF portfolio. This portfolio will reflect each asset class with rules to either increase (overweight) or decrease (underweight) investments based on the VIX and Variance Premiums levels. The aim is to design a strategy that will benefit from the relationships between the VIX and various asset classes, as analyzed in Part I. By focusing on this straightforward yet effective approach, we aim to create a model that balances market potential and risk management, providing a robust solution for private investors seeking to navigate volatile markets.

3.1 Strategy and Portfolio Construction

3.1.1 Overlay strategy

An overlay strategy is an advanced technique used to control risk and add potential for higher returns on a portfolio. It can use derivatives like futures, options, swaps or ETFs without the need to directly adjust the underlying investments. This method is especially useful for hedging against market fluctuations or optimizing the portfolio's position (Chen 2023).

3.1.1.1 Types of Overlay Strategies

3.1.1.1.1 Futures Contracts

- **Pros:** They provide significant leverage without large capital outlays and allow for quick portfolio adjustments.

- **Cons:** Leverage can magnify losses, and futures have expiration dates, leading to additional costs. More generally, derivatives need to be rolled periodically, incurring fees and requiring regular management.

3.1.1.1.2 Options Contracts

- **Pros:** Options offer strategy flexibility with defined risk levels and can be used for specific risk hedging.
- **Cons:** They may incur high costs due to premiums, particularly in volatile markets, and can be complex.

3.1.1.1.3 Swaps

- **Pros:** Swaps offer customizable exchanges of various financial instruments for flexible investment strategies.
- **Cons:** They carry counterparty risk and can be complex and less liquid, making them harder to reverse.

3.1.1.1.4 Exchange-Traded Funds (ETFs)

- **Pros:** These provide straightforward exposure to a wide array of assets, are highly liquid, and transparent.
- **Cons:** They may, sometimes, suffer from tracking errors or lower liquidity and come with management fees that impact returns.

3.1.2 DAA Strategy with ETFs

Considering our objectives, an ETF-only portfolio stands out as the ideal method for implementing Volatility-based DAA strategies due to various factors (Fidelity Not dated):

- **Efficiency in Adjusting Exposure:** ETFs enable rapid adjustments to asset allocations without the need to directly buy and sell the underlying assets.
- **Cost-Effectiveness:** ETFs generally have lower transaction costs compared to the use of derivatives. Lower transaction costs are crucial for a strategy that may require frequent rebalancing in response to volatility levels changes.
- **Liquidity Management:** ETFs are known for their high liquidity. This liquidity ensures that large positions can be entered or exited without significantly impacting the market price, which is essential for maintaining the flexibility required in DAA strategies.
- **Simplification of the R Model:** Using ETFs simplifies the computational and logistical aspects of the DAA models in R. By dealing with a limited number of ETFs as opposed to a vast array of individual assets, the models can be streamlined, enhancing both its performance and maintainability.

3.1.3 ETF Portfolio Constructions – Considerations & Chosen ETFs

In the exploration of DAA strategies, we crafted a tailored ETF-based portfolio model. The portfolio is built on a foundation of ETFs with over a decade of performance history, underscoring their resilience and long-term viability. This model exhibits strategic diversification in terms of asset classes.

The constructed portfolio offers a straightforward mix of asset classes, ideal for investors seeking broad market coverage with simplicity in mind. While our simulations do not account for transaction fees, their omission is unlikely to significantly impact the overall performance given the inherent cost-efficiency of ETFs.

3.1.3.1 Base Portfolio – Broad Asset Classes

It is imperative to consider that the first US ETF was introduced to markets in 1993. ETFs took time before starting to grow in popularity, it was therefore a hard work to find ETFs with long enough track record to constitute the model portfolio (D. Simpson 2024) and be able to execute our backtest within our in-sample period.

In this sense, we have also adjusted the in-sample and out-sample periods considering the following:

- In-sample period : From 01.01.2008 to 31.12.2018
- Out-sample period : From 01.01.2019 to 31.12.2023

In comparison to the various descriptive statistical analyses and estimations executed in Part I and II, only the in-sample period has been curtailed.

The out-sample period remains the same within all parts of this thesis. It will only be used as a final backtest of the strategies prior to drawing conclusions.

Cash & Fixed Income:

- **SPDR Bloomberg 1-3 Month T-Bill ETF (Yahoo Finance Not dated) :** The fund invests at least 80% of its assets in securities of the index, which measures the performance of U.S. Treasury obligations with maturities between 1 and 3 months.
- **iShares $ Corp Bond UCITS ETF (iShares Not dated):** This ETF aims to track the performance of the Bloomberg Barclays USD Corporate Bond Index (Bloomberg Not dated) (including USD Investment Grade corporate bonds).

Equities:

- **SPDR S&P 500 ETF Trust (State Street Not dated) :** The most well-known ETF. It tracks the performance of the S&P 500 Index.
- **Vanguard Information Tech. Index Fund ETF (Vanguard Not dated) :** This ETF aims to track the performance of the MSCI US Investable Market Information Technology 25/50 Index (MSCI Not dated). Information Technology is a cyclical industry which includes many high beta stocks that benefit from low volatility market conditions.

Commodities & Precious Metals:

- **WisdomTree Broad Commodities ETF (WisdomTree Not dated) :** This fund aims to provide exposure to a broad range of commodities, including energy, metals, and agricultural products.
- **SPDR Gold Shares (SPDR Not dated) :** One of the largest gold-backed ETFs, it aims to track the price of gold bullion[17].

3.1.4 Base Portfolio Allocation and Benchmark

3.1.4.1 Portfolio allocation – Market Stability

A balanced approach is typically designed to achieve a moderate risk profile by diversifying investments across various asset classes.

The goal of such approach is to strike a harmony between risk and return, capitalizing on the growth potential of equities while mitigating risk through bonds and other stable assets (Fernando 2022).

The asset allocation of the initial model portfolio during market stability has a balanced approach (Fernando 2022) with below consideration:

- Cash : 5%
- Fixed Income : 35%
- Equities : 55%
- Commodities and Precious Metals : 5%

[17] “Bullion” refers to physical gold bars or coins made for investment purposes.

3.1.4.2 Benchmark Considerations

To ensure consistency in our strategy and analysis, we will use the ***Vanguard Wellington Admiral Fund*** (Yahoo Finance Not dated) as our benchmark. This fund generally maintains a 60/40 allocation (Quantstart Not dated) between equities and bonds, with 60% to 70% mostly invested in dividend-paying common stocks of large companies and 30% to 40% in fixed income securities, including investment-grade corporate bonds, U.S. Treasury, government agency bonds, and mortgage-backed securities.

The use of this benchmark is thoughtful and strategic for several reasons:

- **Balanced Risk and Return:** The 60/40 allocation ensures a moderate risk profile with the potential for steady growth and income. This provides a clear contrast for evaluating the effectiveness of our DAA strategies. Furthermore, it is aligned with our "*Market Stability*" allocation.

- **Consistency and Comparability:** By maintaining a consistent benchmark, we can accurately assess the performance of our Volatility-based DAA strategies across different market regimes. This comparison highlights the added value and effectiveness of our DAA strategies in outperforming a stable, traditional allocation.

The ***Vanguard Wellington Admiral Fund***, with its rebalancing and diversified allocation, serves as an effective benchmark to evaluate our DAA strategies.

Furthermore, our "*Market Stability*" allocations sets (see sections 3.3.2.1.2 and 3.3.2.2.1) will be used as additional benchmarks during our in and out-sample backtests. The benchmarks are detailed and described under section 3.3.2.3.

3.2 Volatility Thresholds Definition

3.2.1 VIX Levels

The initial step in rolling out the new DAA is to set up the strategy that will passively dictate the new allocation. From the analyses in Part I, we have established the following interpretations for VIX levels and their associated market conditions (levels considerations used in practice by majority of practitioners (Fidelity International 2019)) :

- Below 12 indicates « Low Volatility »

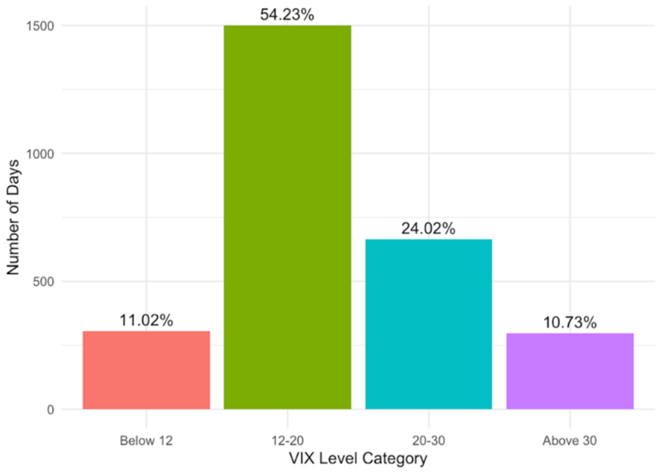
- 12 to 20 suggests « Market Stability »

- 20 to 30 signals « High Volatility »

- Above 30 points to « Very High Volatility »

We simplify the model by not accounting for the premium of 4 to 5 percentage points the VIX typically carries over actual volatility (Lehtonen 2023).

Figure 18 - VIX Levels Distribution (In-Sample)



The above table shows that more than 50% of the time (during the in-sample period – detailed in the following sections), the VIX level has been in the “Market Stability” mode – Indicating that the market was mostly in a moderate volatility state.

The VIX has been in extreme modes (Low Volatility and Very High Volatility) 21.75% of the time during the in-sample period.

As a result, our primary focus will be on capturing performance during extreme volatility periods, given that our portfolios will typically operate within the balanced "Market Stability" mode. The flexibility of our DAA strategies will be evaluated to determine their effectiveness in achieving performance during these extreme volatility phases.

3.2.2 Variance Premiums Levels

The below descriptive statistics have been done to be able to assign Variance Premium (as introduced in Part II of this thesis) levels for each VIX Category.

In Part II of this thesis, we have defined the Variance Premium as the difference between the implied volatility (the VIX) and the historical realized volatility, also called realized volatility (rolling volatility). The implied volatility, represented by the VIX, reflects market expectations of future volatility. Historical realized volatility (rolling volatility), on the other hand, measures the actual volatility of an asset based on its past price movements and indicates how much the asset's price has fluctuated over a specific period.

The Variance Premium is calculated by subtracting the realized volatility from the VIX. This difference is considered to hold the anticipatory element of future market movements. Following our analyzes in Part II, it is interesting to define Variance Premium levels calculated with the 252-day rolling volatility (named "252-Day VP") and the 21-day rolling volatility (named "21-Day VP") and add these volatility metrics as factors of re-balancing within our strategies, to see how these compare with our strategies based on the VIX levels.

We will now analyze the Variance Premium (during the in-sample period) to define levels for our DAA strategy. This analysis will help us determine appropriate thresholds and allocations based on the calculated Variance Premiums levels. The below table shows the mean and standard deviation of each calculated Variance Premium in the various volatility modes, already set by the VIX levels analyzed in the previous sections.

Table 6 - Descriptive Statistics – Variance Premium Levels (In-Sample)

VIX Category	**Mean 252-day VP**	**S. D. 252-day VP**	**Mean 21-day VP**	**S. D. 21-day VP**
Low Volatility (VIX>12)	1.27	2.05	3.96	1.85
Market Stability (VIX=12-20)	0.958	3.33	3.74	3.53
High Volatility (VIX=20-30)	0.0653	10.5	4.74	4.59
Very High Volatility (VIX>30)	6.16	11.7	4.39	7.13

21-day Variance Premium : The mean of the 21-day VP remains relatively stable across different VIX categories, ranging from 3.74 to 4.74. This suggests that short-term realized volatility expectations are very correlated to VIX levels and of similar levels (as already seen in Part II).

252-day Variance Premium : The mean of the 252-day VP shows greater variability, particularly at very high VIX levels (above 30), where it spikes to 6.16. This indicates that the VIX responds more rapidly to changes in volatility, whereas the 252-day VP takes longer to adjust to the same levels. In contrast, the 21-day VP, which is highly correlated with the VIX, adjusts more quickly. This lag in adjustment explains why the values are higher for the 252-day VP compared to the 21-day VP.

The below boxplots visually represent the distribution of the Variance Premiums across different VIX categories during the in-sample period:

Figure 19 - Variance Premium (21d) by VIX Categories (In-Sample)

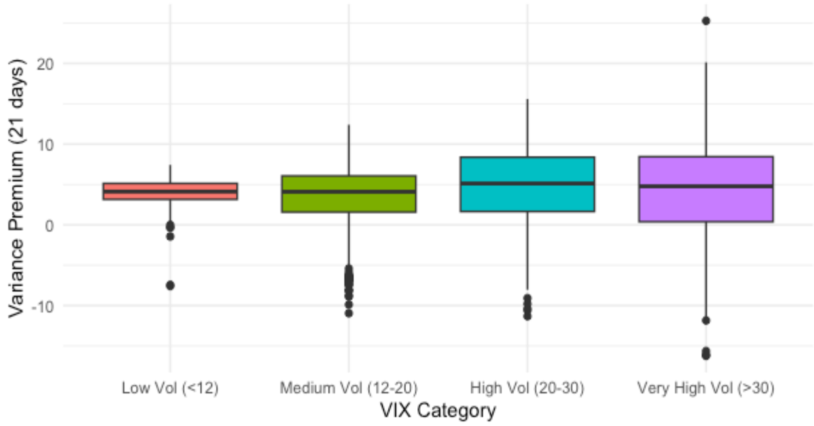
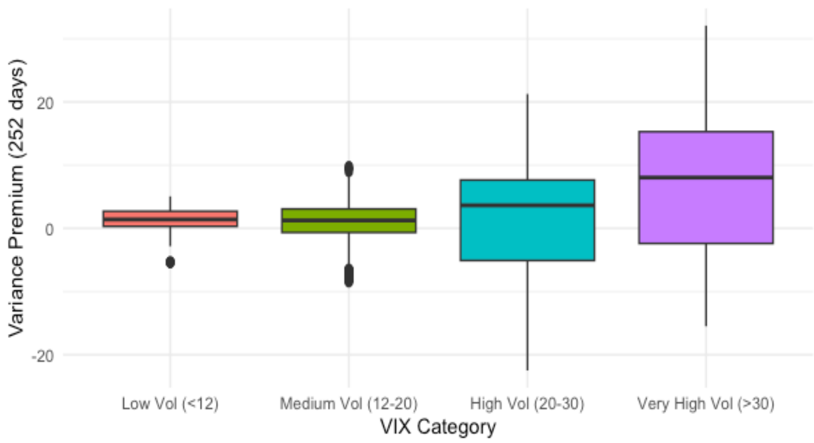


Figure 20 - Variance Premium (252d) by VIX Categories (In-Sample)



Figures 19 and 20 above reveal patterns that help to better understand the relationship between VIX levels and the variance premium.

The 21-day VP boxplots show significant variability across different VIX categories. The median values remaining stable, there is a clear indication of the high correlation between the 21-day VP and the VIX levels.

In contrast, the 252-day VP displays more stability, especially in the lower and medium VIX levels. Considering this analysis, the 252-day VP appears to be a prudent indicator, as its average and volatility increase with VIX levels. This alignment indicates that the 252-day VP may be a better indicator than the VIX for longer-term volatility trends.

For the 252-day VP, the thresholds levels were chosen based on the distribution of the Variance Premiums and their corresponding mean and standard deviation values across different VIX categories. Since the most correlated indicators have been selected (the boxplots increase with VIX levels), it indicates that both indicators (252-day VP and VIX level) are attempting to measure the same phenomena, the recent changes in volatility :

Table 7 - 252-day Variance Premium Levels (In-Sample)

Low Volatility	***Market Stability***	***High Volatility***	***Very High Volatility***
< 0	**0 - 3**	**3 – 6**	**> 6**

On the other hand, the results of the 21-day VP levels show a relative independence from the VIX regimes, as indicated by the similar boxplots across all VIX categories. This suggests that the 21-day VP is attempting to measure a different element.

Given the lack of significant results from the initial boxplots for the 21-day VP, a different approach is employed by defining levels based on quartiles. This method provides a statistically significant categorization. The quartile thresholds are as follows:

- ***25% : 1.389908***
- ***50% : 3.921691***
- ***75% : 6.071395***

Table 8 - 21-day Variance Premium Levels (In-Sample)

Low Volatility	***Market Stability***	***High Volatility***	***Very High Volatility***
< ***1.389908***	***1.389908*** **-** **3.921691**	**3.921691 - 6.071395**	**> 6.071395**

3.3 Backtest (In-Sample)

3.3.1 Asset Allocation Model and Decisions

Based on the VIX and Variance Premium levels outlined in the previous sections, our allocation rules are defined as follows:

Table 9 - Market Conditions vs. Model Portfolio Re-Balancing

Market conditions	**Overweight / Underweight**
Low volatility	Equities may be overweighted (Segal 2021), particularly sectors with higher beta, such as Technology.
Market stability	Maintain a balanced approach across asset classes. This is a neutral market condition where no major reallocations might be necessary, as the existing distribution should already cater to market stability.
High volatility	Fixed Income may be overweighted.
Very high volatility	Fixed Income may be overweighted while Equities may be underweighted (Chen 2022c).

More specifically, below are the exact asset allocation rules for each asset class and in detail, for each underlying (ETF) constituting our initial allocations set named **MONTBLANC**:

Table 10 - Market Conditions vs. Model Portfolio Re-Balancing (Detailed)

Asset	Low Volatility (VIX < 12)	Medium Volatility (VIX 12-20)	High Volatility (VIX 20-30)	Very High Volatility (VIX > 30)
SPDR Bloomberg 1-3 Month T-Bill ETF (BIL)	0.00%	5.00%	5.00%	0.00%
iShares $ Corp Bond UCITS ETF (LQDE.L)	0.00%	35.00%	50.00%	70.00%
SPDR S&P 500 ETF Trust (SPY)	80.00%	50.00%	40.00%	25.00%
Vanguard Information Tech. Index Fund ETF (VGT)	15.00%	5.00%	0.00%	0.00%
WisdomTree Broad Commodities ETF (AIGC.L)	2.00%	2.00%	2.00%	2.00%
SPDR Gold Shares (GLD)	3.00%	3.00%	3.00%	3.00%

3.3.2 In-sample Backtest

3.3.2.1 Analysis of our Foundations

The in-sample backtest process involves a detailed analysis of various metrics using R. To begin, it was necessary to define our VIX and Variance Premiums levels thresholds and the corresponding portfolio weights. This initial step was crucial as it allowed us to allocate these levels to the portfolio and examine the results across several key metrics.

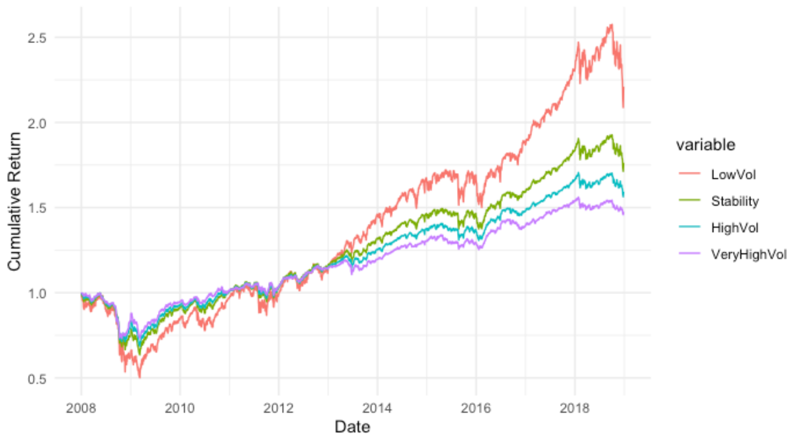
We will now execute few analyses to test the foundation of our model. It will help us identify and address any significant issues or inconsistencies that could impact our backtest results if not considered from the outset. This step is vital as it ensures that our model is robust and reliable before applying it to real-world scenarios.

Through this meticulous examination, we aim to gain valuable insights into how our portfolio performs under different market conditions dictated by varying VIX levels. The process will not only reinforce the validity of our model but also highlight areas where adjustments might be needed. This careful and systematic approach will allow us to gain confidence in the results and enhance the overall credibility of our backtest.

3.3.2.1.1 *Cumulative Returns of Each Allocation*

Each allocation (see Table 10) has been tested as a standalone portfolio throughout the in-sample period. The goal was to evaluate each allocation individually and compare the key metrics.

Figure 21 - Cumulative Returns of Different Allocations (In-Sample)



- **Low Volatility allocation:** Exhibits the highest annualized return (7.46%) but also the highest annualized standard deviation (19.13%). This indicates a more aggressive allocation that captures higher returns with greater risk.

- **Market Stability allocation:** Provides a moderate annualized return (5.29%) with a significantly lower standard deviation (10.92%). This balance suggests a more stable approach with a moderate risk-return profile.

- **High Volatility allocation:** Shows a lower annualized return (4.34%) and a further reduced standard deviation (8.38%). This allocation appears to be more conservative, aiming for stability in volatile markets.

- **Very High Volatility allocation:** Delivers the lowest annualized return (3.60%) with the lowest standard deviation (6.74%). This conservative allocation minimizes risk during periods of very high volatility.

3.3.2.1.2 *Descriptive Statistics*

Table 11 - Initial Allocations Results Analyses (In-Sample)

Initial Allocations	Low Volatility (VIX < 12)	Market Stability (VIX 12-20)	High Volatility (VIX 20-30)	Very High Volatility (VIX > 30)
Annualized Return	7.46%	5.29%	4.34%	3.60%
Annualized Std Dev	19.13%	10.92%	8.38%	6.74%
Annualized Sharpe (Rf=0.42%)	0.366	0.4436	0.4647	0.4694
Mean	0.04%	0.02%	0.02%	0.01%
Standard Deviation	1.21%	0.69%	0.53%	0.42%
Skewness	0.1357	-0.1655	-0.4671	-1.1085
Kurtosis	13.2314	11.054	12.0447	18.5756

The various calculated metrics indicate that the risk-adjusted return improves as the allocation becomes more conservative in response to higher VIX level considerations. The **Sharpe ratio** increases as the VIX level consideration goes up – from 0.3660 (for Low Volatility allocation) up to 0.4694 (for the Very High Volatility allocation).

- **Mean Return:** Decreases from Low Volatility to Very High Volatility, reflecting the lower expected returns in more conservative portfolios.
- **Standard Deviation:** Declines progressively, consistent with the reduction in risk-taking. It is normal for the standard deviation to vary inversely with market volatility; in Low Volatility mode, the portfolio overweights the most volatile ETFs, while in Very High Volatility regime, it holds very low volatility products such as cash and bonds.
- **Skewness and Kurtosis:** Skewness becomes more negative, and kurtosis increases, particularly in the Very High Volatility allocation. This suggests a higher probability of extreme negative returns in this very conservative allocation, indicating more pronounced tail risk.

3.3.2.1.3 *Drawdown Analysis*

Figure 22 - Drawdown of Various Allocations (In-Sample)

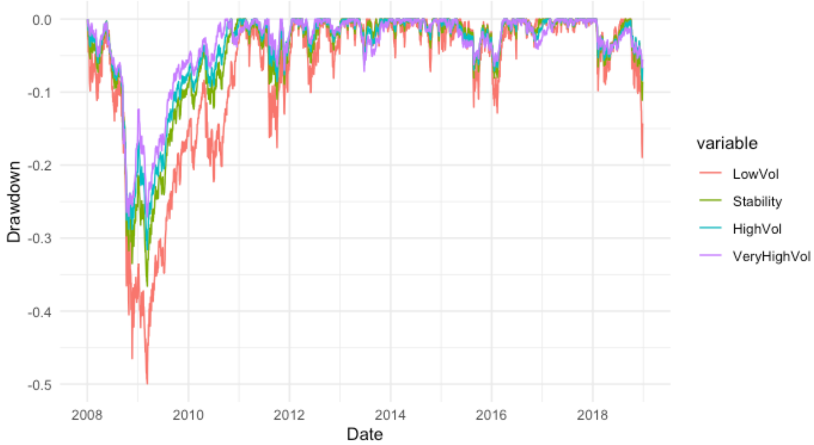


Figure 22 presents the drawdown analysis conducted on each allocation :

- **Low Volatility allocation:** Experiences the highest maximum drawdown (49.94%), reflecting its higher risk exposure.
- **Market Stability allocation:** Has a lower maximum drawdown (36.59%), indicating better risk management.
- **High Volatility allocation:** Further reduces maximum drawdown (31.57%), showing increased resilience.
- **Very High Volatility allocation:** Achieves the lowest maximum drawdown (27.10%), highlighting its effectiveness in protecting capital.

3.3.2.1.4 Results Analysis and Conclusion

Based on the above analyses, our current allocations set provide a range of risk and return profiles that are suitable for different volatility profiles, as required in building our DAA strategies. However, the results suggest few points that require further consideration :

Low Volatility allocation: Exhibits the highest annualized return (7.46%) but also the highest annualized standard deviation (19.13%). Experiences the highest maximum drawdown (49.94%), reflecting its higher risk exposure.

Market Stability allocation: Provides a moderate annualized return (5.29%) with a significantly lower standard deviation (10.92%). Has a lower maximum drawdown (36.59%), indicating better risk management.

High Volatility allocation: Shows a lower annualized return (4.34%) and a further reduced standard deviation (8.38%). Further reduces maximum drawdown (31.57%), showing increased resilience.

Very High Volatility allocation: Delivers the lowest annualized return (3.60%) with the lowest standard deviation (6.74%). Achieves the lowest maximum drawdown (27.10%), highlighting its effectiveness in protecting capital during extreme volatility.

3.3.2.2 New Allocations Set

Given the results of the initial allocations set, it is beneficial to test another set in the aim to strike a better balance between capturing market potential and cautious risk management. As explained since the start of this thesis, the aim of our model is to create a passive DAA. In a portfolio management context, the risk metrics are important to take into consideration. Therefore, we have tested various combinations and analyzed various results. The bellow allocations are the ones that provide the better results in minimizing risk.

The adjustments were made mainly considering risk management ; The new allocations set aims to reduce overall portfolio volatility by lowering the standard deviation, providing more steady returns and minimizing large fluctuations in value, which is crucial for risk-averse investors. Additionally, the improved maximum drawdown offers better protection against significant losses during market downturns, preserving capital and maintaining investor confidence during high volatility periods.

New allocations set (named ZERMATT):

Table 12 - Market Conditions vs. New Model Portfolio Re-Balancing (Detailed)

Asset	Low Volatility (VIX < 12)	Medium Volatility (VIX 12-20)	High Volatility (VIX 20-30)	Very High Volatility (VIX > 30)
SPDR Bloomberg 1-3 Month T-Bill ETF (BIL)	10.00%	10.00%	15.00%	20.00%
iShares $ Corp Bond UCITS ETF (LQDE.L)	15.00%	30.00%	45.00%	50.00%
SPDR S&P 500 ETF Trust (SPY)	60.00%	45.00%	30.00%	15.00%
Vanguard Information Tech. Index Fund ETF (VGT)	7.50%	5.00%	2.00%	0.00%
WisdomTree Broad Commodities ETF (AIGC.L)	3.00%	5.00%	5.00%	7.50%
SPDR Gold Shares (GLD)	4.50%	5.00%	3.00%	7.50%

The above allocations set have an increased focus on diversification within each volatility range with higher allocation to cash, bonds, commodities and precious metals buckets mainly.

3.3.2.2.1 Results Analysis and Conclusion

Table 13 - New Allocations Results Analyses (In-Sample)

New Allocations	Low Volatility (VIX < 12)	Market Stability (VIX 12-20)	High Volatility (VIX 20-30)	Very High Volatility (VIX > 30)
Annualized Return	5.80%	4.69%	3.52%	2.26%
Annualized Std Dev	13.38%	10.15%	7.05%	5.16%
Annualized Sharpe (Rf=0.42%)	0.3997	0.4187	0.4378	0.3548
Mean Return	0.03%	0.02%	0.01%	0.01%
Standard Deviation	0.84%	0.64%	0.44%	0.32%
Skewness	-0.0149	-0.1867	-0.5517	-0.9088
Kurtosis	11.3719	10.3107	11.3364	13.5422
Max Drawdown	40.11%	34.69%	27.88%	21.67%

The comparison between initial and new allocations set reveal a shift towards a more conservative approach in the new allocations set, as intended. This change results in:

- **Lower annualized returns across all volatility levels:** This is expected as the new allocations set prioritizes risk management and stability over aggressive growth.

- **Reduced standard deviation, indicating lower risk:** The new allocations set achieves a significant reduction in volatility, making the portfolios less susceptible to large swings in value.

- **Improved maximum drawdown figures, suggesting better protection against significant losses:** The focus on more stable assets and a higher allocation to bonds and gold has effectively reduced the maximum drawdowns.

Overall, the new allocations set provides a more stable and risk-averse strategy, which is particularly beneficial during high volatility periods. However, this comes at the cost of lower returns. The adjustments made in the new allocations set aim to strike a better balance between capturing market potential and managing risk. By reducing the exposure to high-risk assets and increasing allocations to more stable investments, the new allocations set offers a more conservative approach that helps to safeguard the portfolio against extreme market conditions.

3.3.2.3 Market Stability in Montblanc and Zermatt: Additional benchmarks

As stated earlier in Part III, we have decided to have two new benchmarks, in addition to the ***Vanguard Wellington Admiral Fund***.

To be able to further compare our models, we have also decided to include the *Market Stability* allocations of **Montblanc** and **Zermatt** allocations sets as new benchmarks in our models. This will allow us to also compare our models results to our fixed *Market Stability* allocations and see whether reallocations based on volatility metrics enhance returns and risk metrics.

Table 14 - M- Benchmark & Z-Benchmark (In-Sample)

ALLOCATIONS	M-BENCHMARK	Z-BENCHMARK
CASH	5.00%	10.00%
BONDS	35.00%	30.00%
EQUITIES	50.00%	45.00%
EQUITIES (TECH)	5.00%	5.00%
COMMO.	2.00%	5.00%
PM (GOLD)	3.00%	5.00%

RESULTS	M-BENCHMARK	Z-BENCHMARK
Annualized Return	5.29%	4.69%
Annualized Std Dev	10.92%	10.15%
Annualized Sharpe (Rf=0.42%)	0.4436	0.4187

3.3.2.4 In-Sample Backtest – Considerations

Based on all the analyses conducted, we will proceed with the in-sample backtest of the various portfolios on our two allocations set (initial allocations set, named ***MONTLANC***, and new allocations set, named ***ZERMATT***). This backtest will include the following strategies:

Table 15 - In-Sample Backtest Strategies

BASIC STRATEGIES

MONTBLANC BASIC ALLOCATIONS				
Asset	Low Volatility	Market Stability	High Volatility	Very High Volatility
CASH	0.00%	5.00%	5.00%	0.00%
BONDS	0.00%	35.00%	50.00%	70.00%
EQUITIES	80.00%	50.00%	40.00%	25.00%
EQUITIES (TECH)	15.00%	5.00%	0.00%	0.00%
COMMO.	2.00%	2.00%	2.00%	2.00%
PM (GOLD)	3.00%	3.00%	3.00%	3.00%

ZERMATT BASIC ALLOCATIONS				
Asset	Low Volatility	Market Stability	High Volatility	Very High Volatility
CASH	10.00%	10.00%	15.00%	20.00%
BONDS	15.00%	30.00%	45.00%	50.00%
EQUITIES	60.00%	45.00%	30.00%	15.00%
EQUITIES (TECH)	7.50%	5.00%	2.00%	0.00%
COMMO.	3.00%	5.00%	5.00%	7.50%
PM (GOLD)	4.50%	5.00%	3.00%	7.50%

VIX Levels	
Low Volatility	<12
Market Stability	12-20
High Volatility	20-30
Very High Volatility	>30

PEAK STRATEGIES

MONTBLANC PEAK ALLOCATIONS				
Asset	Low Volatility	Market Stability	High Volatility	Very High Volatility
CASH	0.00%	5.00%	5.00%	0.00%
BONDS	0.00%	35.00%	50.00%	70.00%
EQUITIES	80.00%	50.00%	40.00%	25.00%
EQUITIES (TECH)	15.00%	5.00%	0.00%	0.00%
COMMO.	2.00%	2.00%	2.00%	2.00%
PM (GOLD)	3.00%	3.00%	3.00%	3.00%

ZERMATT PEAK ALLOCATIONS				
Asset	Low Volatility	Market Stability	High Volatility	Very High Volatility
CASH	10.00%	10.00%	15.00%	20.00%
BONDS	15.00%	30.00%	45.00%	50.00%
EQUITIES	60.00%	45.00%	30.00%	15.00%
EQUITIES (TECH)	7.50%	5.00%	2.00%	0.00%
COMMO.	3.00%	5.00%	5.00%	7.50%
PM (GOLD)	4.50%	5.00%	3.00%	7.50%

252-day VP Levels	
Low Volatility	<0
Market Stability	0-3
High Volatility	3-6
Very High Volatility	>6

EDGE STRATEGIES

MONTBLANC EDGE ALLOCATIONS				
Asset	Low Volatility	Market Stability	High Volatility	Very High Volatility
CASH	0.00%	5.00%	5.00%	0.00%
BONDS	0.00%	35.00%	50.00%	70.00%
EQUITIES	80.00%	50.00%	40.00%	25.00%
EQUITIES (TECH)	15.00%	5.00%	0.00%	0.00%
COMMO.	2.00%	2.00%	2.00%	2.00%
PM (GOLD)	3.00%	3.00%	3.00%	3.00%

ZERMATT EDGE ALLOCATIONS				
Asset	Low Volatility	Market Stability	High Volatility	Very High Volatility
CASH	10.00%	10.00%	15.00%	20.00%
BONDS	15.00%	30.00%	45.00%	50.00%
EQUITIES	60.00%	45.00%	30.00%	15.00%
EQUITIES (TECH)	7.50%	5.00%	2.00%	0.00%
COMMO.	3.00%	5.00%	5.00%	7.50%
PM (GOLD)	4.50%	5.00%	3.00%	7.50%

21-day VP Levels	
Low Volatility	<1.389908
Market Stability	1.389908-3.921691
High Volatility	3.921691-6.071395
Very High Volatility	>6.071395

The **BASIC** strategies, ***MONTBLANC BASIC*** and ***ZERMATT BASIC***, are centered around VIX levels. These strategies will be evaluated based on how they perform when reallocations are executed upon VIX levels, which has been the foundational idea that set the direction for this thesis.

The **PEAK** strategies, ***MONTBLANC PEAK*** and ***ZERMATT PEAK***, are based on the Variance Premium levels calculated from rolling 252-day realized volatility. As explained previously, the use of the 252-day VP can help identify significant shifts in long-term volatility, allowing us to understand whether this can serve as a better anticipatory volatility indicator than the VIX itself. Being less correlated with the VIX, it can serve as a confirmation signal for long-term trends and can be beneficial for reducing the impact of short-term fluctuations, leading to potentially smoother performance. The longer rolling window averages out short-term noise, providing a stable measure of market expectations and enhancing the reliability of the indicator for long-term investment decisions.

The **EDGE** strategies, ***MONTBLANC EDGE*** and ***ZERMATT EDGE*** are based on the Variance Premium levels calculated from rolling 21-day realized volatility. The use of the 21-day VP could be suitable for strategies that need to respond quickly to recent market changes. As it is highly correlated with the VIX, it makes the indicator effective for short-term adjustments. This correlation allows for rapid response to volatility spikes, capturing immediate market movements, and adjusting allocations swiftly, which can lead to higher short-term gains.

3.3.3 In-Sample Backtest – Results Analysis

3.3.3.1 Strategies based on the VIX levels – BASIC Strategies

Table 16 - Key Metrics of BASIC Strategies (In-Sample)

Metric	MONTBLANC BASIC Strategy	ZERMATT BASIC Strategy	Wellington	S&P 500
Annualized Return	4.80%	3.69%	6.55%	7.21%
Annualized Standard Dev.	8.08%	6.98%	12.19%	20.07%
Annualized Sharpe (Rf=0.42%)	0.5392	0.4654	0.5004	0.3365

Performance Overview:

- **Annualized Returns:** The Wellington Fund outperformed both MONTBLANC BASIC and ZERMATT BASIC in terms of raw returns. MONTBLANC BASIC had higher returns than ZERMATT BASIC.

- **Annualized Sharpe Ratio:** MONTBLANC BASIC demonstrated the best risk-adjusted performance, outperforming both ZERMATT BASIC and the Wellington Fund. Although ZERMATT BASIC had the lowest Sharpe ratio, it still provided reasonable risk-adjusted returns.

Key Findings:

- MONTBLANC BASIC offered a balanced approach with good returns and moderate risk, achieving the best risk-adjusted returns.
- ZERMATT BASIC was more conservative, focusing on minimizing volatility at the expense of lower returns.
- The Wellington Fund balanced higher returns with higher risk.

In summary, the MONTBLANC BASIC strategy stood out for its superior risk-adjusted returns. The ZERMATT BASIC strategy excelled in minimizing volatility. The Wellington Fund offered a high return with higher risk.

3.3.3.2 Strategies based on the 252-day Variance Premium – PEAK Strategies

Table 17 - Key Metrics of the PEAK Strategies (In-Sample)

Metric	MONTBLANC PEAK Strategy	ZERMATT PEAK Strategy	Wellington
Annualized Return	5.25%	3.73%	6.55%
Annualized Standard Dev.	10.22%	7.31%	12.19%
Annualized Sharpe (Rf=0.42%)	0.4705	0.4511	0.5004

Performance Overview:

- **Annualized Returns:** The Wellington Fund achieved the highest annualized return, followed by MONTBLANC PEAK. ZERMATT PEAK had the lowest return among the three.
- **Annualized Sharpe Ratio:** The Wellington Fund demonstrated the best risk-adjusted returns with the highest Sharpe ratio. MONTBLANC PEAK had a slightly higher Sharpe ratio than ZERMATT PEAK, indicating better risk-adjusted returns.

Key Findings:

- The MONTBLANC PEAK offered higher returns with moderate risk, achieving decent risk-adjusted returns.
- The ZERMATT PEAK provided a more conservative approach with lower volatility and lower returns.
- The Wellington Fund balanced higher returns with higher risk, outperforming both PEAK strategies.

The Wellington Fund stood out for its superior risk-adjusted returns and highest raw returns. The ZERMATT PEAK excelled in minimizing volatility. The MONTBLANC PEAK offered higher returns with moderate risk, providing a balanced approach. This comparison highlights the effectiveness of the PEAK strategies in managing risk while achieving stable performance, though both underperformed compared to the Wellington Fund.

3.3.3.3 Strategies based on the 21-day Variance Premium – EDGE Strategies

Table 18 - Key Metrics of the EDGE Strategies (In-Sample)

Metric	MONTBLANC EDGE Strategy	ZERMATT EDGE Strategy	Wellington
Annualized Return	0.48%	1.51%	6.55%
Annualized Standard Dev.	10.70%	9.98%	12.19%
Annualized Sharpe (Rf=0.42%)	0.0051	0.108	0.5004

Performance Overview:

- **Annualized Returns:** The Wellington Fund significantly outperformed both MONTBLANC EDGE and ZERMATT EDGE in terms of annualized returns. ZERMATT EDGE had a higher return than MONTBLANC EDGE.
- **Annualized Sharpe Ratio:** The Wellington Fund demonstrated the best risk-adjusted returns with the highest Sharpe ratio. ZERMATT EDGE had a better Sharpe ratio compared to MONTBLANC EDGE, indicating superior risk-adjusted performance.

Key Findings:

- The MONTBLANC EDGE offered the lowest returns and risk-adjusted returns, with moderate volatility.
- The ZERMATT EDGE provided a more conservative approach with lower volatility and better risk-adjusted returns compared to MONTBLANC EDGE.
- The Wellington Fund balanced higher returns with higher risk, outperforming both EDGE strategies.

The Wellington Fund stands out for its superior risk-adjusted returns and highest raw returns. The ZERMATT EDGE shows promise with lower volatility and better risk-adjusted performance than the MONTBLANC EDGE.

However, both EDGE strategies underperformed the Wellington Fund with poor performance and high standard deviation, highlighting the need for further refinement.

3.3.4 Key Takeaways and Considerations for the Out-Sample Backtest

Table 19 - Key Metrics of All Strategies (In-Sample)

Metric	BASIC - VIX		PEAK - 252d VP		EDGE - 21d VP	
	MONTBLANC	ZERMATT	MONTBLANC	ZERMATT	MONTBLANC	ZERMATT
Ann.Return	4.80%	3.69%	5.25%	3.73%	0.48%	1.51%
Ann. Standard Dev.	8.08%	6.98%	10.22%	7.31%	10.70%	9.98%
Annu. Sharpe	0.5392	0.4654	0.4705	0.4511	0.0051	0.108

Benchmarks	*Wellington*	*M-BENCHMARK*	*Z-BENCHMARK*
Ann.Return	6.55%	5.29%	4.69%
Ann. Standard Dev.	12.19%	10.92%	10.15%
Annu. Sharpe	0.5004	0.4436	0.4187

BASIC Strategies Using VIX: The BASIC strategies underperform the benchmarks in terms of returns but manage to provide higher Sharpe ratios, indicating better risk-adjusted returns. This suggests that the VIX-based allocations effectively manage risk but do not fully capture market gains. When considering the number of days with re-allocations, table 20 shows that BASIC strategies had on average 169 days within the in-sample period.

PEAK Strategies Using 252-day Variance Premium: The PEAK strategies yield the highest returns and competitive Sharpe ratios, indicating strong performance both in absolute and risk-adjusted terms. Table 20 indicates that PEAK strategies averaged 335 days of re-allocation.

EDGE Strategies Using 21-day Variance Premium: The EDGE strategies show lower returns than the other strategies, with higher standard deviations, indicating increased volatility. Table 20 shows that EDGE strategies had on average 526 days with portfolio re-balancing.

Table 20 - Number of Re-Allocations per Strategy (In-Sample)

Strategy	Total days	Mean
MONTBLANC BASIC	143	**168.5**
ZERMATT BASIC	194	
MONTBLANC PEAK	304	**334.5**
ZERMATT PEAK	365	
MONTBLANC EDGE	483	**526**
ZERMATT EDGE	569	

The in-sample backtest results highlight the strengths and weaknesses of different strategies. The PEAK strategies using the 252-day VP outperform in terms of returns and offer competitive risk-adjusted performance, demonstrating their effectiveness in predicting future volatility and providing stable adjustments. The BASIC strategies using VIX manage to provide higher Sharpe ratios, indicating good risk management despite lower returns. On the other hand, the EDGE strategies using the 21-day VP exhibit higher volatility and lower risk-adjusted returns, suggesting that these strategies need refinement.

Overall, the results support the use of the 252-day VP for a more stable and effective DAA strategy.

3.3.5 Optimization Models

3.3.5.1 Optimization Model on Allocations

It is important to note that our base model operates on the premise of fixed allocations across different volatility regimes. The shift from one allocation to another within each strategy is solely determined by the volatility factor level. Specifically, the BASIC allocations shift based on VIX levels, the PEAK allocations on 252-day VP levels, and the EDGE allocations on 21-day VP levels. For each strategy, we have defined two sets of allocations through the various volatility regimes (initial allocations set named MONTBLANC and new allocations set named ZERMATT).

This methodology has been consistently applied throughout all our in-sample backtest so far, with the following optimization model suggesting new optimal allocations set for each strategy.

3.3.5.1.1 Methodology – Function In R

In R, we have designed a function to find the optimal asset allocation across strategies, based on varying volatility signals (VIX, VP 252-day, VP 21-day).

The function starts by segmenting the data of each strategy. For each strategy, key statistical parameters are computed :

- **Arithmetic mean return:** Average return for each asset class (in our case, each asset class is defined by a single ETF – Except for equities, for which we have S&P500 (SPY) and tech. equities exposure (VGT)).
- **Standard deviation:** Volatility of returns for each asset class.
- **Variance-Covariance matrix:** Relationship between the returns of the different asset classes.

By using the above parameters, the efficient frontier is calculated. The efficient frontier represents a set of optimal portfolios offering the highest expected return for a defined level of risk. The Sharpe ratio is calculated for each portfolio on the efficient frontier. The portfolio with the highest Sharpe ratio (called the Tangent Portfolio[18]) is identified.

Then, the function returns the asset class weights of the tangent portfolio (rounded and adjusted to ensure they sum to one – to ensure we do not have leveraged portfolio in case the sum is above one, or have cash left in our portfolio if the sum is below one (in our portfolios, we have a specific ETF considered for the cash bucket, therefore all the assets have to be invested)). This result is the optimal asset allocation for each strategy.

3.3.5.1.2 Optimal Asset Allocations Sets for Each Strategy

Below are the results of optimal asset allocations sets determined by our function.

BASIC Optimal Allocations:

Table 20 - Optimal Allocations Set – BASIC Strategy (In-Sample)

OPTIMAL ALLOCATION - BASIC STRATEGY	Low Volatility (VIX < 12)	Medium Volatility (VIX 12-20)	High Volatility (VIX 20-30)	Very High Volatility (VIX > 30)
SPDR Bloomberg 1-3 Month T-Bill ETF (BIL)	15.00%	0.00%	90.00%	45.00%
iShares $ Corp Bond UCITS ETF (LQDE.L)	20.00%	70.00%	5.00%	0.00%
SPDR S&P 500 ETF Trust (SPY)	25.00%	0.00%	0.00%	0.00%
Vanguard Information Tech. Index Fund ETF (VGT)	15.00%	30.00%	0.00%	30.00%
WisdomTree Broad Commodities ETF (AIGC.L)	0.00%	0.00%	0.00%	0.00%
SPDR Gold Shares (GLD)	25.00%	0.00%	5.00%	25.00%

The BASIC optimal allocations set maintains a relatively balanced diversification across multiple asset classes including cash, bonds, equities (SPY and VGT) and precious metals. It aims to provide stability and growth, keeping a mix of assets.

PEAK Optimal Allocations:

Table 21 - Optimal Allocations Set – PEAK Strategy (In-Sample)

OPTIMAL ALLOCATION - PEAK STRATEGY	Low Volatility (252VP < 0)	Medium Volatility (252VP 0-3)	High Volatility (252VP 3-6)	Very High Volatility (252VP >6)
SPDR Bloomberg 1-3 Month T-Bill ETF (BIL)	5.00%	0.00%	90.00%	85.00%
iShares $ Corp Bond UCITS ETF (LQDE.L)	70.00%	50.00%	0.00%	0.00%
SPDR S&P 500 ETF Trust (SPY)	0.00%	0.00%	5.00%	5.00%
Vanguard Information Tech. Index Fund ETF (VGT)	25.00%	35.00%	0.00%	0.00%
WisdomTree Broad Commodities ETF (AIGC.L)	0.00%	0.00%	0.00%	0.00%
SPDR Gold Shares (GLD)	0.00%	15.00%	5.00%	10.00%

The PEAK optimal allocations set focuses heavily on cash and bonds, particularly in Low Volatility and Very High Volatility modes. It shows a strong preference for income-generating and stable assets, reducing exposure to equities and commodities.

[18] The tangent portfolio is the portfolio lying at the point where the efficient frontier is tangent to the highest capital market line. It offers the highest Sharpe ratio.

EDGE Optimal Allocations:

Table 22 - Optimal Allocations Set – EDGE Strategy (In-Sample)

OPTIMAL ALLOCATION - EDGE STRATEGY	Low Volatility (21VP < 1.39)	Medium Volatility (21VP 1.39-3.92)	High Volatility (21VP 3.92-6.07)	Very High Volatility (21VP >6.07)
SPDR Bloomberg 1-3 Month T-Bill ETF (BIL)	0.00%	95.00%	5.00%	0.00%
iShares $ Corp Bond UCITS ETF (LQDE.L)	95.00%	0.00%	80.00%	0.00%
SPDR S&P 500 ETF Trust (SPY)	0.00%	0.00%	0.00%	0.00%
Vanguard Information Tech. Index Fund ETF (VGT)	5.00%	5.00%	0.00%	75.00%
WisdomTree Broad Commodities ETF (AIGC.L)	0.00%	0.00%	5.00%	0.00%
SPDR Gold Shares (GLD)	0.00%	0.00%	10.00%	25.00%

The EDGE optimal allocation is the most concentrated and aggressive in its allocation shifts. In Low Volatility mode, it invests almost entirely in bonds. As volatility increases, it shifts dramatically towards equities (VGT only) and precious metals.

3.3.5.1.3 Results Analysis and Conclusion

The differences between the BASIC, PEAK and EDGE optimal allocations highlight varying portfolio management philosophies. The BASIC optimal allocation emphasizes a balanced diversification, the PEAK optimal allocation focuses on capital preservation with a conservative approach and the EDGE optimal allocation seeks high returns through aggressive allocation shifts.

Table 23 - Key Metrics of the Optimal Allocations Sets Strategies (In-Sample)

OPTIMAL ALLOCATIONS	BASIC	PEAK	EDGE
Annualized Return	5.65%	6.89%	11.82%
Annualized Standard Dev.	6.34%	5.21%	12.25%
Annualized Sharpe (Rf=0.42%)	0.822	1.2351	0.9264

The in-sample backtest of the defined optimal allocations set for each strategy show interesting findings :

- **Return comparison :** The EDGE optimal allocations set significantly outperforms the other allocations, demonstrating the potential of aggressive allocation strategies to capture high returns.
- **Risk comparison :** The PEAK optimal allocations set is the least volatile. The EDGE optimal allocations set, while providing the highest returns, comes with the highest volatility.
- **Sharpe Ratio :** The PEAK optimal allocations set leads in risk-adjusted performance, making it the most efficient in balancing return and risk.

The results from the in-sample backtest support the optimal allocations proposed by the model. They align well with the theoretical expectations for each strategy.

3.3.5.2 Optimization Model on Thresholds

3.3.5.2.1 Methodology – Function In R

As in our previous optimization model detailed in section 3.3.5.1, we have designed a function to identify the best thresholds for our volatility metrics, which are central to our in-sample backtest. These thresholds allow us to reallocate our portfolio and rebalance asset weights during our defined volatility regimes: Low Volatility, Market Stability, High Volatility, and Very High Volatility.

In this optimization model, we only optimize the thresholds while keeping the allocations fixed. These fixed allocations are our initial allocations set, named **MONTLANC**, and our new allocations set, named **ZERMATT**. The optimization aims to find the portfolio that maximizes the Sharpe ratio, offering the best risk-adjusted return:

- The function begins with thresholds set to zero and a very low baseline Sharpe ratio. This serves as a starting point for comparison.
- It systematically examines each combination of probabilities, which are quantiles derived from our volatility metric vectors.
- For each set of thresholds, the function calculates the annualized Sharpe ratio. It compares each calculated Sharpe ratio with the maximum Sharpe ratio found so far. If a new Sharpe ratio is higher, the corresponding thresholds are updated as the optimal ones.

The combination of thresholds that yields the highest Sharpe ratio represents the optimal volatility thresholds. By maximizing the Sharpe ratio, this function ensures that our portfolio achieves the best possible risk-adjusted return.

3.3.5.2.2 Optimal Thresholds for Each Volatility Metric

Table 24 - Optimal Thresholds – BASIC and PEAK Strategies (In-Sample)

MONTBLANC BASIC	VIX LEVELS	ZERMATT BASIC	VIX LEVELS
Low Volatility	<14.02	**Low Volatility**	<15.46
Market Stability	14.02-14.68	**Market Stability**	15.46-16.91
High Volatility	14.68-15.46	**High Volatility**	16.91-21.19
Very High Volatility	>15.46	**Very High Volatility**	>21.19

MONTBLANC PEAK	252-DAYS VP	ZERMATT PEAK	252-DAYS VP
Low Volatility	<0.862	**Low Volatility**	<1.76
Market Stability	0.862-1.295	**Market Stability**	1.76-4.41
High Volatility	1.295-2.212	**High Volatility**	4.41-5.48
Very High Volatility	>2.212	**Very High Volatility**	>5.48

Both the VIX and 252-day VP thresholds for ZERMATT are higher and cover broader ranges compared to MONTBLANC. This adjustment likely aims to capture more market conditions within each volatility regime. The ZERMATT allocations had previously been designed to enhance the Sharpe ratio. In this context, the broader and higher thresholds for ZERMATT are intended to better differentiate between varying levels of market volatility and stability, allowing for more precise and effective asset reallocation.

Also, it is important to note that the optimization does not include the 21-day VP levels, as it was initially based on quartiles. Therefore, the EDGE Strategies are not taken into consideration for the optimized thresholds.

3.3.5.2.3 Results Analysis and Conclusion

Table 25 - Key Metrics of the Optimal Thresholds Strategies (In-Sample)

OPTIMAL THRESHOLDS	BASIC		PEAK	
	MONTBLANC	ZERMATT	MONTBLANC	ZERMATT
Annualized Return	5.99%	4.22%	6.30%	5.00%
Annualized Standard Dev.	8.18%	6.42%	9.96%	8.14%
Annualized Sharpe (Rf=0.42%)	0.6776	0.5883	0.5877	0.56

The in-sample backtest results indicate that while the ZERMATT allocations were successful in reducing volatility and managing risk, the MONTBLANC allocations delivered higher raw and risk-adjusted returns.

The MONTBLANC BASIC strategy stood out for its superior Sharpe ratio, demonstrating the best risk-adjusted performance.

3.3.5.3 Out-Sample Backtest Considerations

Optimal Allocations:

For the out-sample backtest, we will implement the optimal allocations sets as fixed allocations sets throughout the out-sample period. This approach implies that by fixing the optimal allocations, the results obtained in the in-sample backtest could significantly vary, if the market dynamics throughout the out-sample period shift significantly.

Optimal Thresholds:

For the out-sample backtest, we will use fixed optimal thresholds, similar to our approach in our in-sample backtest (see sections 3.3.2.3 to 3.3.4), while maintaining fixed allocations for both MONTBLANC (initial allocations set) and ZERMATT (new allocations set).

3.4 Backtest (Out-Sample)

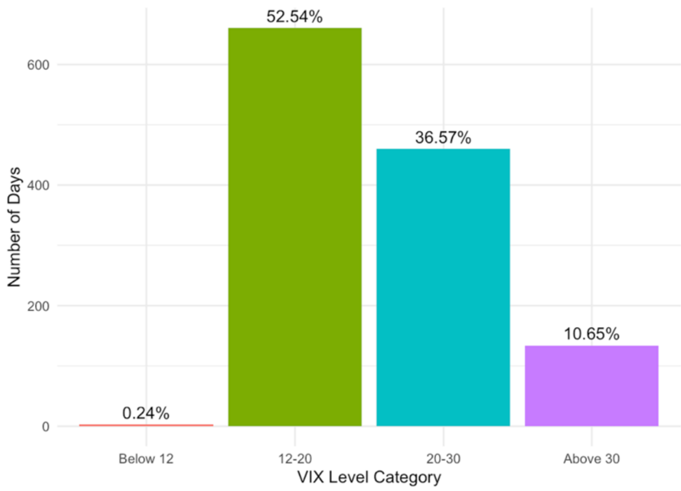
As our final step, we will conduct out-sample backtest to validate our strategies and draw meaningful conclusions. This involves testing the strategies[19] on data from **January 1, 2019 to December 31, 2023**, which was not used during the in-sample testing phase. This ensures that our strategies are robust and unbiased.

3.4.1 Volatility Metrics Behavior During the Out-Sample Period

Before analyzing the results of our strategies during the out-sample period, it is necessary to examine the behavior of our volatility metrics (VIX levels, 252-day VP, and 21-day VP) to understand the implications of the fixed thresholds used during the in-sample backtest.

3.4.1.1 VIX Levels

Figure 23 - VIX levels Distribution (Out-Sample)



The table shows that more than 50% of the time, similar to the in-sample period, the VIX level was in the “Market Stability” mode, indicating a moderate volatility state. The VIX was in extreme modes (Low Volatility and Very High Volatility) 10.89% of the time during the out-sample period, compared to 21.75% during the in-sample period. Notably, the “High Volatility” mode increased from 24.02% in the in-sample period to 36.57% in the out-sample period, indicating a rise of over 50%.

[19] Please refer to Table 14 under section 3.3.5.1 “*In-Sample Backtest: Considerations*”

3.4.1.2 Variance Premiums Levels

Table 26 - Descriptive Statistics – Variance Premium Levels (Out-Sample)

VIX Category	Mean 252-day VP	S. D. 252-day VP	Mean 21-day VP	S. D. 21-day VP
Low Volatility (VIX>12)	*n.a.*	*n.a.*	*n.a.*	*n.a.*
Market Stability (VIX=12-20)	-0.478	3.80	4.34	3.16
High Volatility (VIX=20-30)	0.485	7.80	4.78	5.38
Very High Volatility (VIX>30)	13.3	12.5	0.161	15.8

The Low Volatility mode was nearly nonexistent (0.24% of the time), making results for this category not applicable. However, significant changes in Variance Premiums between the in-sample and out-sample periods highlight shifts in market dynamics :

Market Stability Category:

- The mean for the 252-day VP is negative (-0.478) in the out-sample period, contrasting with the positive mean premium in the in-sample period (0.958).
- The mean for the 21-day VP increased slightly from 3.74 (in-sample) to 4.34 (out-sample).

High Volatility Category:

- The mean for the 252-day VP increased in the out-sample period (0.485) compared to the in-sample period (0.0653).
- The mean for the 21-day VP remained stable with a slight increase from 4.74 (in-sample) to 4.78 (out-sample).
- The standard deviation increased in the out-sample period for both the 252-day and 21-day VPs.

Very High Volatility Category:

- The mean for the 252-day VP increased significantly in the out-sample period (13.3) compared to the in-sample period (6.16).

- The mean for the 21-day VP decreased dramatically from 4.39 (in-sample) to 0.161 (out-sample).

- The standard deviation for the 21-day VP increased substantially, indicating higher variability in extreme market conditions.

3.4.1.3 Implications

We observe a general tendency in VIX levels across both periods, but substantial changes in Variance Premiums could lead to significantly different outcomes in our out-sample strategy backtests.

3.4.1 Out-Sample Strategies Backtest – Results Analysis

We will now compare the performance of our different strategies between the in-sample and out-sample period. For the out-sample backtest, we consider the strategies *MONTBLANC BASIC*, *ZERMATT BASIC*, *MONTBLANC PEAK*, *ZERMATT PEAK*, *MONTBLANC EDGE* and *ZERMATT EDGE* and the benchmarks Wellington, M-Benchmark, Z-Benchmark.

Table 27 - Key Metrics of All Strategies (Out-Sample)

	OUTSAMPLE BACKTEST					
Metric	**BASIC - VIX**		**PEAK - 252d VP**		**EDGE - 21d VP**	
	MONTBLANC	**ZERMATT**	**MONTBLANC**	**ZERMATT**	**MONTBLANC**	**ZERMATT**
Ann.Return	7.18%	7.19%	12.71%	9.33%	8.34%	5.97%
Ann. Standard Dev.	11.59%	9.60%	14.41%	10.03%	15.14%	12.10%
Annu. Sharpe	0.4489	0.5433	0.7378	0.729	0.4187	0.3315

	OUTSAMPLE BACKTEST		
Benchmarks	***Wellington***	***M-BENCHMARK***	***Z-BENCHMARK***
Ann.Return	9.65%	10.75%	10.41%
Ann. Standard Dev.	13.61%	13.83%	12.71%
Annu. Sharpe	0.5603	0.6291	0.6585

The Wellington Fund (one of our benchmarks) performed well in both in-sample and out-sample periods. It achieved an annual return of 6.55% in-sample, increasing to 9.65% out-sample. The standard deviation rose from 12.19% to 13.61%, but the Sharpe ratio also improved from 0.5004 to 0.5603. It maintained strong returns and managed risk effectively despite increased market volatility.

Our constructed benchmarks (M-Benchmark and Z-Benchmark) performed also well in both in-sample and out-sample periods. They both achieved stronger annual returns out-sample. The standard deviations also increased but the Sharpe ratios improved significantly.

Annualized Return:

- **MONTBLANC BASIC:** In-sample return was 4.80%, while out-sample return improved to 7.18%. This increase can be attributed to the overall market recovery and significant growth phases in the post-COVID period.
- **ZERMATT BASIC:** In-sample return was 3.69%, while out-sample return increased to 7.19%. The out-sample period presented more opportunities due to market volatility, especially during the COVID-19 pandemic and subsequent recovery.
- **MONTBLANC PEAK:** In-sample return was 5.25%, significantly increasing to 12.71% out-sample. This substantial increase is likely due to higher market volatility providing more opportunities for 252-day VP.
- **ZERMATT PEAK:** In-sample return was 3.73%, rising to 9.33% out-sample. The higher returns during the out-sample period suggest that the strategy adapted well to the increased volatility.
- **MONTBLANC EDGE:** In-sample return was 0.48%, increasing to 8.34% out-sample. The considerable increase indicates that 21-day VP strategies were particularly effective during the out-sample period.
- **ZERMATT EDGE:** In-sample return was 1.51%, increasing to 5.97% out-sample.

Annualized Sharpe Ratio:

- **MONTBLANC BASIC:** Decreased from 0.5392 (in-sample) to 0.4489 (out-sample). Despite higher returns, the increased volatility reduced the Sharpe ratio.
- **ZERMATT BASIC:** Increased from 0.4654 (in-sample) to 0.5433 (out-sample). This suggests a better risk-adjusted performance during the out-sample period.
- **MONTBLANC PEAK:** Increased from 0.4705 (in-sample) to 0.7378 (out-sample). The strategy's performance improved significantly on a risk-adjusted basis.
- **ZERMATT PEAK:** Increased from 0.4511 (in-sample) to 0.729 (out-sample). This indicates a substantial improvement in risk-adjusted returns.
- **MONTBLANC EDGE:** Increased from 0.0051 (in-sample) to 0.4187 (out-sample). The dramatic improvement shows that the strategy adapted well to the out-sample market conditions.

- **ZERMATT EDGE:** Increased from 0.108 (in-sample) to 0.3315 (out-sample). The increase highlights better performance on a risk-adjusted basis.

Implications:

The comparison between in-sample and out-sample backtests reveal several key insights:

MONTBLANC PEAK emerged as the best-performing strategy with the highest annual return (12.71%) – above all benchmarks – and Sharpe ratio (0.7378) in the out-sample period. This strategy significantly benefited from increased market volatility, allowing it to capitalize on the use of 252-day VP.

MONTBLANC BASIC and ZERMATT BASIC showed solid improvements in annual returns, but the increased volatility resulted in mixed variations to their Sharpe ratios. MONTBLANC BASIC saw a decrease, while ZERMATT BASIC saw an increase, indicating better risk-adjusted performance for ZERMATT BASIC.

EDGE strategies, especially MONTBLANC EDGE, showed substantial improvements in both returns and Sharpe ratio, indicating that the 21-day VP strategies were highly effective during the out-sample period's volatile market conditions – However the substantial increase of the results highlights the instability and volatility of these strategies.

Overall, the strategies that focused on Variance Premiums, particularly the PEAK strategies, performed very well, leveraging the heightened market volatility from 2019 to 2023 to achieve higher returns and better risk-adjusted performance compared to the benchmark.

3.4.2 Out-Sample Optimized Strategies Backtest – Results Analysis

We will now analyze the out-sample backtest results of our optimized strategies, which have been executed both on the asset allocations set for each strategy and the thresholds volatility levels for the considered metrics in the BASIC (VIX) and PEAK (252-day VP) strategies.

3.4.2.1 Optimized Asset Allocations Sets

Table 28 - Key Metrics of the Optimal Allocations Strategies (Out-Sample)

OUTSAMPLE BACKTEST			
OPTIMAL ALLOCATIONS	**BASIC**	**PEAK**	**EDGE**
Annualized Return	9.33%	12.78%	12.28%
Annualized Standard Dev.	7.92%	7.48%	15.35%
Annualized Sharpe (Rf=0.42%)	0.923	1.4298	0.6648

The optimized PEAK strategy emerged as the best-performing strategy in terms of return and risk-adjusted performance. Its ability to adapt to varying volatility conditions allowed it to leverage opportunities effectively, resulting in the highest annualized return (12.78%) and Sharpe ratio (1.4298). As described in section 3.3.5.1.2, the PEAK optimal allocations set strongly focus on cash and bonds, showing a strong preference for income-generating and stable assets, which proved beneficial during the out-sample period.

The optimized BASIC strategy showed improved returns and a better Sharpe ratio out-sample, indicating robust performance. However, it lagged behind the PEAK strategy in terms of overall efficiency. As described in section 3.3.5.1.2, the BASIC optimal allocations set maintains a relatively balanced diversification across assets (aiming for stability).

The optimized EDGE strategy continued to deliver high returns, but its increased volatility led to a decline in the Sharpe ratio. This suggests that while the EDGE strategy can capture substantial returns, it comes with higher risk. As described in section 3.3.5.1.2, the EDGE optimal allocations set are the most concentrated and aggressive in their allocation shifts. In the Very High Volatility regime, the fund allocates 75% to tech. equities and 25% to gold.

Overall, the PEAK strategy with optimized allocations set stands out as the most efficient in balancing return and risk, making it the best choice for capitalizing on market conditions.

The backtest results demonstrate strong robustness, as the optimized allocation sets consistently deliver favorable metrics (returns and risk) out-of-sample. This is uncommon, as optimizations often align well with historical data but struggle with future performance, making these results particularly valuable.

However, despite the strong performance both in and out-of-sample, the concentrated allocations suggested by the Variance Premium strategies could pose challenges in private wealth management, where it may be difficult to justify a less diversified portfolio to clients.

3.4.2.2 Optimized Thresholds

Table 29 - Key Metrics of the Optimal Thresholds Strategies (Out-Sample)

OUTSAMPLE BACKTEST				
OPTIMAL THRESHOLDS	**BASIC**		**PEAK**	
	MONTBLANC	**ZERMATT**	**MONTBLANC**	**ZERMATT**
Annualized Return	6.73%	13.96%	6.36%	9.86%
Annualized Standard Dev.	11.62%	14.42%	8.58%	10.11%
Annualized Sharpe (Rf=0.42%)	0.4095	0.8224	0.512	0.7746

BASIC Strategies – VIX levels optimization: Both MONTBLANC BASIC and ZERMATT BASIC showed improved returns, but ZERMATT BASIC achieved a better Sharpe ratio, indicating superior risk-adjusted performance.

PEAK Strategies – 252-day VP levels optimization: The PEAK strategies, particularly ZERMATT PEAK, excelled in balancing high returns with effective risk management, making them optimal choices for capitalizing on market conditions.

Overall, the optimized thresholds allowed the PEAK strategies to achieve the best balance between return and risk, demonstrating their effectiveness in different market environments. The BASIC strategies also performed well, providing stable returns with manageable risk.

3.4.3 Methodology and Key Findings

3.4.3.1 Methodology

The methodology for Part III of the thesis is structured to provide various Volatility-Based DAA strategies models conducted through R models to validate our key findings in Part I and II. It involves the following :

- **Coding methods:** All the subsequent points were executed through R codes which include data manipulations, creation of advanced plots, historical classifications, optimizations, risk analyses, time series analyses, correlation analyses, descriptive statistics, web scraping, functions coding, data handling, and more.
- **Data Analyses:** We defined our VIX levels thresholds based on historical data. We then defined the Variance Premium as a "future-volatility" metric and definition of our Variance Premium indicators levels thresholds (252-day VP and 21-day VP).
- **Literature Review:** We introduced overlay strategies and determined the best way to create our strategies using ETFs, detailed in the related sections.
- **Portfolio Construction:** We selected ETFs to construct the base model and the foundation of our models using R.
- **Further constructions:** We defined two different allocations sets (initial allocations set – MONTBLANC and new allocations set – ZERMATT) for each volatility mode. Selected and defined benchmark and asset allocation decision rules.

- **In-sample backtest:** We evaluated our strategies through in-sample backtest (2 different allocations sets for all defined volatility modes resulting in a total of 8 strategies). We analyzed our final strategies (BASIC strategies – based on VIX levels, PEAK strategies – based on 252-day VP levels, EDGE strategies – based on 21-day VP levels) through our defined allocations sets.

- **Optimization models:** We developed two optimization models. This involved optimizing through allocations sets (defining the optimized allocations sets to replace MONTBLANC and ZERMATT through each strategy : BASIC, PEAK, EDGE) and optimizing volatility metric thresholds (determining when to shift allocations for our strategies: BASIC and PEAK).

- **Out-sample backtest:** We conducted the same analysis as in the in-sample backtest. We set optimized allocations sets and thresholds as fixed variables for the out-sample period to assess if our optimizations are viable within various timeframes.

In conclusion, this methodology offers a comprehensive evaluation of Volatility-Based DAA strategies, using R for detailed data analysis and modeling.

3.4.3.2 Key Findings – Model, Strategies and Optimizations

3.4.3.2.1 Allocations (MONTBLANC vs. ZERMATT)

The **MONTBLANC** allocations set aims to maintain a balanced diversification across multiple asset classes. It focuses on providing stability and growth by keeping a mix of assets such as cash, bonds, equities (SPY and VGT), and precious metals. This allocation was designed to adapt to varying volatility regimes while aiming for higher returns and moderate risk management.

The **ZERMATT** allocations set on the other hand, has a more conservative approach. This approach increases the allocation to cash, bonds, commodities, and precious metals while reducing the exposure to equities, particularly during higher volatility regimes. The goal was to minimize risk and drawdowns, providing a steadier return profile even during market turbulences.

Key Findings:

- **Performance:** MONTBLANC allocations set showed higher returns due to its more aggressive stance during lower volatility periods, while ZERMATT allocations set provided lower but more stable returns.

- **Risk Management:** ZERMATT allocations set did great in minimizing volatility and reducing portfolio volatility, making it suitable for risk-averse investors.
- **Sharpe Ratio:** Despite lower raw returns, ZERMATT's allocations set focus on stability led to competitive risk-adjusted returns, often outperforming MONTBLANC allocations set when looking at this metric specifically.

3.4.3.2.2 Strategies (BASIC vs. ZERMATT vs. EDGE)

The **BASIC** strategies utilized VIX levels to determine asset allocations. This approach aimed to balance risk and return based on market volatility expectations, using the implied volatility of the S&P 500.

The **PEAK** strategies employed the 252-day VP as the volatility indicator, which helped in making longer-term allocation adjustments. By capturing the longer-term trends in market volatility, these strategies aimed to provide stable returns and manage risk more effectively.

The **EDGE** strategies relied on the 21-day VP for more frequent, short-term allocation adjustments. This approach was designed to quickly respond to recent market changes, potentially capturing short-term gains but also introducing higher volatility and risk.

Key Findings:

- **Annualized Returns:** The PEAK strategies typically delivered the highest annualized returns, leveraging longer-term market trends for better performance.
- **Risk Management:** The EDGE strategies, while agile, often resulted in higher portfolio volatility and lower risk-adjusted returns compared to PEAK and BASIC strategies.
- **Adaptability:** BASIC strategies provided a balanced approach, effectively managing risk while achieving moderate returns, providing a stable middle ground.

3.4.3.2.3 Optimizations (Allocations Sets and Volatility Thresholds)

Optimizing allocations sets for each strategy led to significant improvements in risk-adjusted performance. The optimal allocations sets tended to shift more towards bonds and cash during High Volatility periods, while favoring equities during Low Volatility periods. These adjustments ensured better alignment with market conditions, enhancing overall returns and reducing risk.

The optimization of volatility thresholds, especially for the BASIC and PEAK strategies, resulted in refined reallocation levels. These optimized thresholds allowed the strategies to more accurately capture changes in market conditions, leading to improved performance metrics.

For instance, the PEAK strategy with optimized thresholds achieved a higher Sharpe ratio and higher annualized returns by better timing its allocations based on the 252-day VP.

Key Findings:

- **Performance Enhancement:** Both optimizations led to substantial performance improvements in out-sample backtest, validating their effectiveness.
- **Risk-Adjusted Returns:** The PEAK strategy with optimized thresholds consistently provided the best risk-adjusted returns, proving the utility of the 252-day VP.
- **Flexibility and Robustness:** The optimized models demonstrated greater flexibility and robustness, effectively navigating volatility regimes.

4. Conclusion

This thesis has allowed me to apply and deepen my understanding of volatility and asset allocation strategies, putting into practice the knowledge I have acquired during my four-year bachelor's degree.

Recognizing that volatility metrics, particularly the VIX, have been extensively studied by many researchers, I aimed to propose a comprehensive overview of basic statistical models and analysis within this well-studied field.

Critique of Strategies and Backtest:

Our strategies and backtests had few limitations:

- **Sample Periods:** The in-sample period from 2008 to 2018 and the out-sample period from 2019 to 2023 were chosen due to the availability of ETFs with sufficient historical data. Although comprehensive, these periods may not capture all possible market scenarios.
- **Market Dynamics within Sample Periods:** In the 2008 to 2018 (in-sample period), the S&P 500's annualized return of 7.21% reflected substantial growth post-financial crisis, despite episodes of volatility. In the 2019 to 2023 (out-sample period), market recovery of the post-COVID-19 pandemic led to an annualized return of 15.61%. While our strategies performed better out-sample, it is hard to tell if this was due to our models or due to the general market growth.
- **Volatility Metrics:** While the VIX and Variance Premium provided valuable insights, our predictability analysis showed that the VIX's predictive power is limited. The VIX finally does not substantially differ from short-term historical rolling volatility.

  If the Variance Premium is calculated on a short-term historical volatility (as in the 21-day VP), testing such metric involves assessing whether there is predictive information in the small difference between the VIX and the short-term historical volatility. The results we obtained show that it is not straightforward : In-sample results are weak, while out-sample results are more favorable. This strategy may perform well during specific periods but its long-term reliability remains uncertain.

  If the Variance Premium is calculated using short-term volatility, given that the VIX closely mirrors it and that short-term Variance Premium has limited predictive power, the Variance Premium essentially becomes the difference between short-term and long-term historical volatility. In this case, the focus shifts to tracking sudden changes in volatility.

- **Behavioral finance:** Although not addressed in this thesis, behavioral finance is essential to nuance the conclusions, as it demonstrates that investors are not always rational, which inevitably influences market volatility. Many researchers emphasize the fact that markets often overreact to economic fundamentals, with much higher standard deviations for S&P500 price fluctuations compared to company earnings or GDP. This disconnect can be explained by investor emotions.

  These emotions can have high impacts on option prices, which can result in inconsistent VIX valuations and give a distorted picture of volatility during periods of market stress.

Results Conclusion:

The results from our strategies were informative:

- When looking solely at the **allocations** (Montblanc allocations set vs. Zermatt allocations set), we note that Montblanc delivered the higher returns. In terms of risk, Zermatt excelled in minimizing volatility, notably by showing very competitive Sharpe Ratios.

- Regarding the **strategies**:

  - **BASIC Strategy:** Utilizing initial VIX levels, this strategy provides a balanced approach suitable for broad market conditions. It is more sellable due to its simplicity and alignment with widely understood VIX metrics.

  - **PEAK Strategy:** Leveraging the 252-day VP, this strategy achieved the best risk-adjusted performance and demonstrated robustness in volatile markets. It could be further enhanced with advanced statistical tools and is ideal for advanced investors having some technical background.

  - **EDGE Strategy:** Designed to capture short-term gains using the 21-day VP, this strategy proved too volatile and unstable. Our inability to establish refined levels, relying on quartiles instead, indicated its potential shortcomings.

Finally, the **Montblanc PEAK Strategy** (initial allocations set, rebalancing based on the 252-day VP), showed the higher results in both in and out-sample, outperforming all benchmarks during the out-sample period (Wellington Fund, M-Benchmark, Z-Benchmark).

In regards to the optimizations :

- **Allocations set optimizations:** The BASIC allocations set (based on the VIX) is the only one relatively balanced across all asset classes. The optimized allocations sets based on the Variance Premium are particularly aggressive and concentrated in focus.

  The results of the backtest are quite robust as the optimized allocations set tend to offer favorable metrics (returns, risk) out-sample. This is quite rare as optimizations typically fit well with past data but are not well-suited for future data. It therefore provides genuine benefits.

  However, despite the strong performance both in and out-sample, it is important to note that the concentrated allocations proposed for the Variance Premium strategies could be challenging in the context of private wealth management, where selling a less diversified portfolio may be difficult.

- **Volatility metrics optimizations:** The optimizations were only executed on Montblanc BASIC, Montblanc PEAK, Zermatt BASIC and Zermatt PEAK. The EDGE strategies were excluded from the optimization as the results of the descriptive statistics were not satisfactory and initial 21-day VP thresholds were determined by the quartiles.

  The optimized PEAK thresholds achieved the best balance between return and risk out-sample. The optimized BASIC thresholds also performed well, providing stable returns with manageable risk.

Final Thoughts:

The findings highlight the importance of integrating volatility measures to develop more resilient portfolios. This thesis represents only the tip of the iceberg in the vast and complex field of volatility-based DAA strategies and more generally, of volatility as a key metric to incorporate in portfolio management decisions.

Our empirical study revealed that the VIX is mostly composed of realized volatility, and that its predictive power is limited. While it provides insights into market expectations, it does not consistently offer forward-looking accuracy, highlighting the need for additional variables and models to improve predictability.

Our conclusions also emphasize that predicting the future with certainty is an illusion, and adhering to economic fundamentals rather than emotions remains a winning long-term strategy.

Future research could extend historical data ranges, incorporate more sophisticated models, and explore additional volatility metrics to further enhance strategy performance. Incorporating behavioral finance into this research could also provide insights into how investors' emotions have historically influenced implied volatility and options pricing, and how these behaviors translate into modern market dynamics.

Moreover, while this study primarily focused on passive strategies, integrating active decision-making and occasional rebalancing could potentially improve portfolio performance. Combining passive and active strategies could offer a more dynamic and responsive approach, balancing the benefits of stability and adaptability.

In conclusion, this study lays the groundwork for more complex research, potentially incorporating active strategies alongside our volatility measures. This dual approach could provide a more robust framework for managing portfolios in an ever-changing market environment, ultimately leading to more effective and more resilient asset allocation strategies.

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Annex 1 : Tables A01 & A02

A01	SnP500.rdt
Annualized Return	0.0543
Annualized Std Dev	0.1836
Annualized Sharpe (Rf=1.21%)	0.2299

A02	S&P 500 daily returns
Min	-0.094695125
First quartile	-0.003975852
Median	0.000622721
Third quartile	0.005235978
Max	0.109571968
Percentage of returns under -1%	11.89403974
Percentage of returns over 1%	12.10596026
Average returns when under -1%	-0.019829693
Average returns when over 1%	0.017959884
Skewness	-0.364337815
Excess kurtosis	8.996958971
p-value Jarque Test	22722.03844

Annex 2 : Tables A04 & A05

A04	x
VIX	1
X0	0.915
X1	0.92
X2	0.923
X3	0.924
X4	0.925
X5	0.924
X6	0.922
X7	0.92
X8	0.916
X9	0.911
X10	0.905
X11	0.897
X12	0.889
X13	0.88
X14	0.87
X15	0.86
X16	0.85
X17	0.839
X18	0.826
X19	0.813
X20	0.799
X21	0.782
X22	0.767
X23	0.754
X24	0.741
X25	0.729
X26	0.718
X27	0.708
X28	0.697
X29	0.687
X30	0.677
X31	0.668
X32	0.66
X33	0.652
X34	0.644
X35	0.637
X36	0.629
X37	0.622
X38	0.615
X39	0.609
X40	0.602

A05	term	estimate	std.error	statistic	p.value
1	(Intercept)	6.970516079	0.100736771	69.19534987	0
2	RollingVol.21	0.758869941	0.005465076	138.8580638	0